

VHE Pulsed Emissions from Rotation-Powered Pulsar

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Crab nebula: Composite image of X-ray [blue] and optical [red]

§1 γ -ray Pulsar Observations

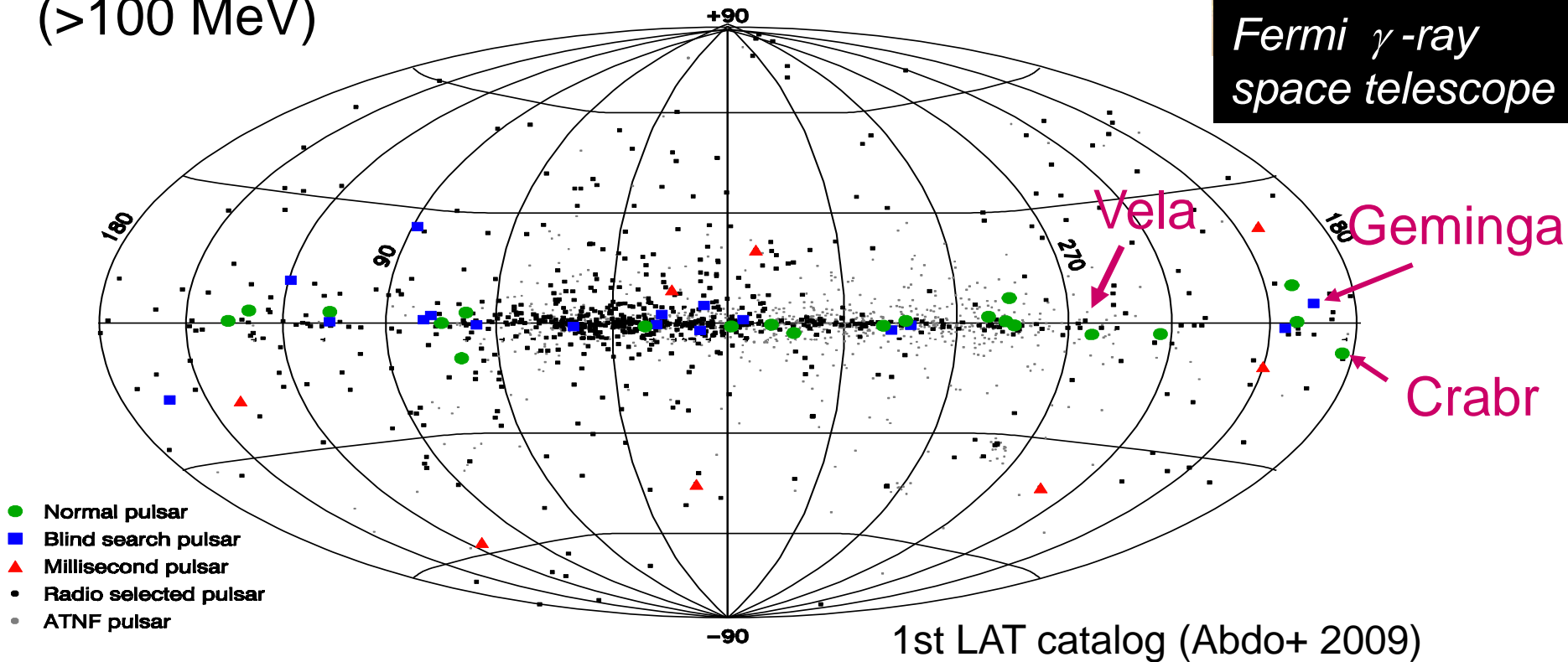
After 2008, LAT aboard Fermi has detected more than **121** pulsars above 100 MeV.

Fermi/LAT point sources
(>100 MeV)



Large Area
Telescope

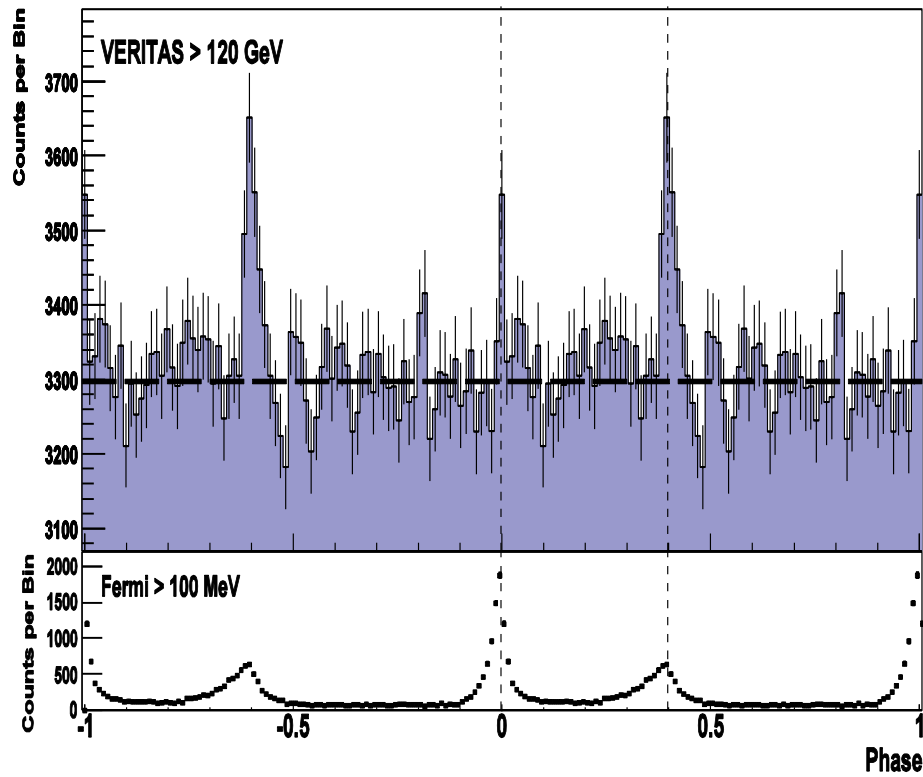
*Fermi γ -ray
space telescope*



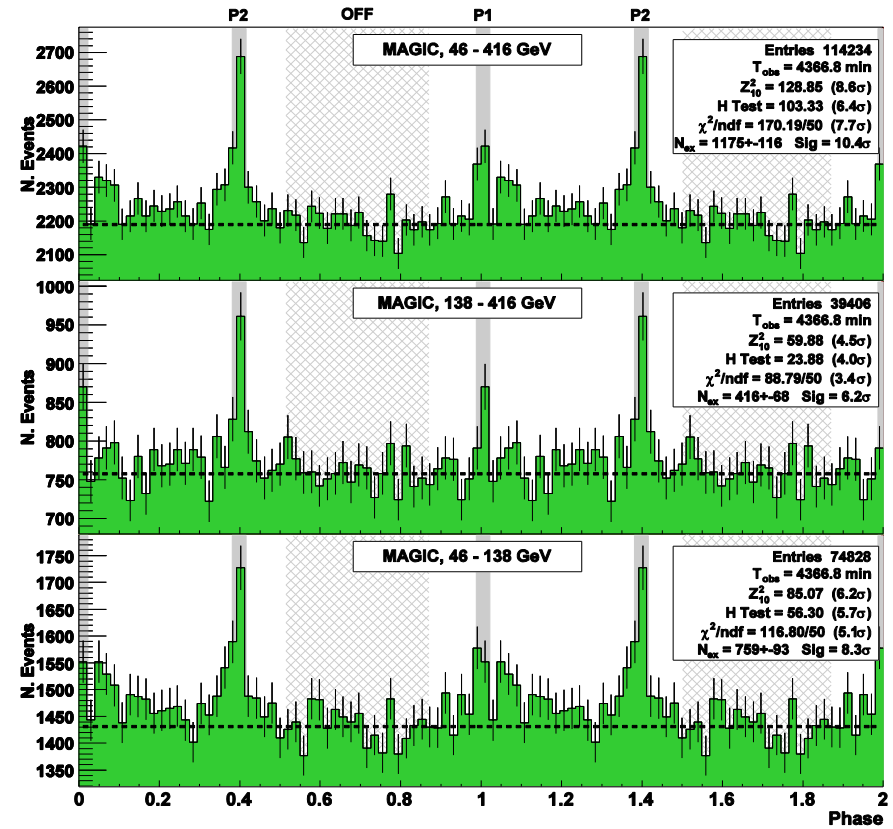
§1 *γ -ray Pulsar Observations*

Recent IACTs found pulsed emission in 25-400 GeV from the Crab pulsar.

VERITAS (> 120 GeV)
Aliu+ (2011, Science 334, 69)



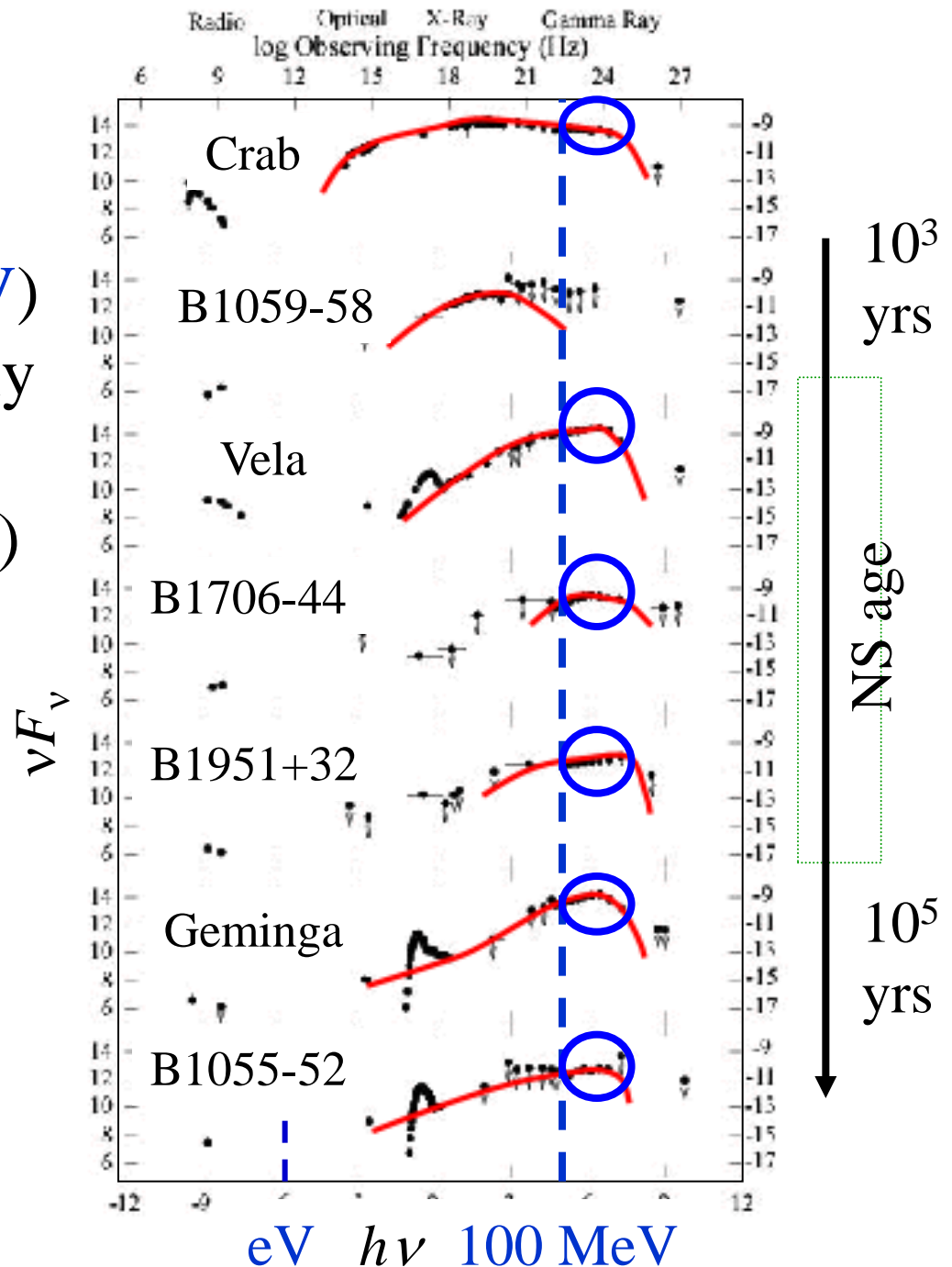
MAGIC (25–416 GeV)
Aleksić+ (2011a,b)



Broad-band spectra (pulsed)

● High-energy ($> 100\text{MeV}$) photons are emitted mainly via **curvature process** by **ultra-relativistic** ($\sim 10\text{TeV}$) e^\pm 's accelerated in pulsar magnetosphere.

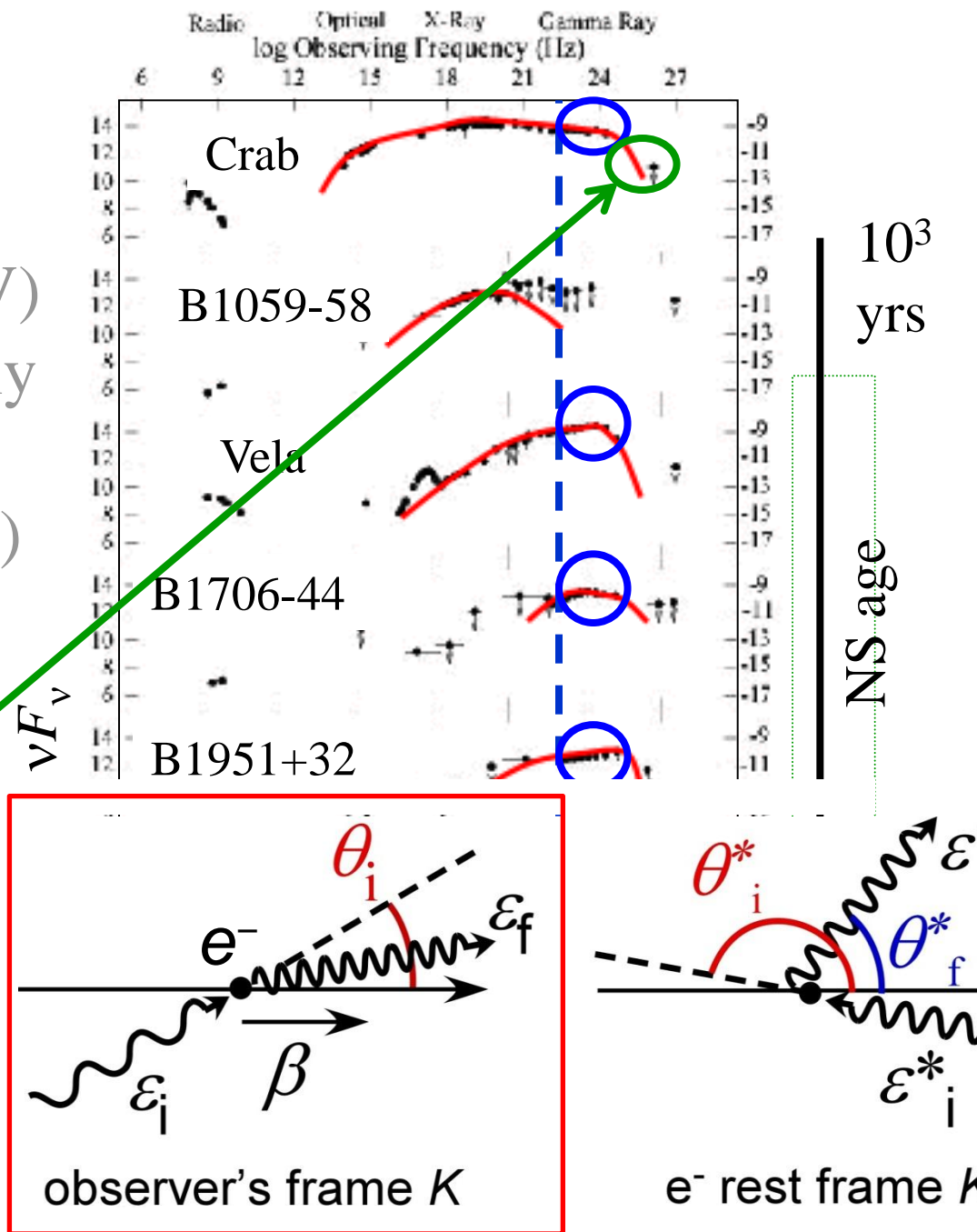
● Above 20 GeV, ICS by secondary/tertiary pairs contributes.



Broad-band spectra (pulsed)

● High-energy ($> 100\text{MeV}$) photons are emitted mainly via **curvature** process by ultra-relativistic ($\sim 10\text{TeV}$) e^\pm 's accelerated in pulsar magnetosphere.

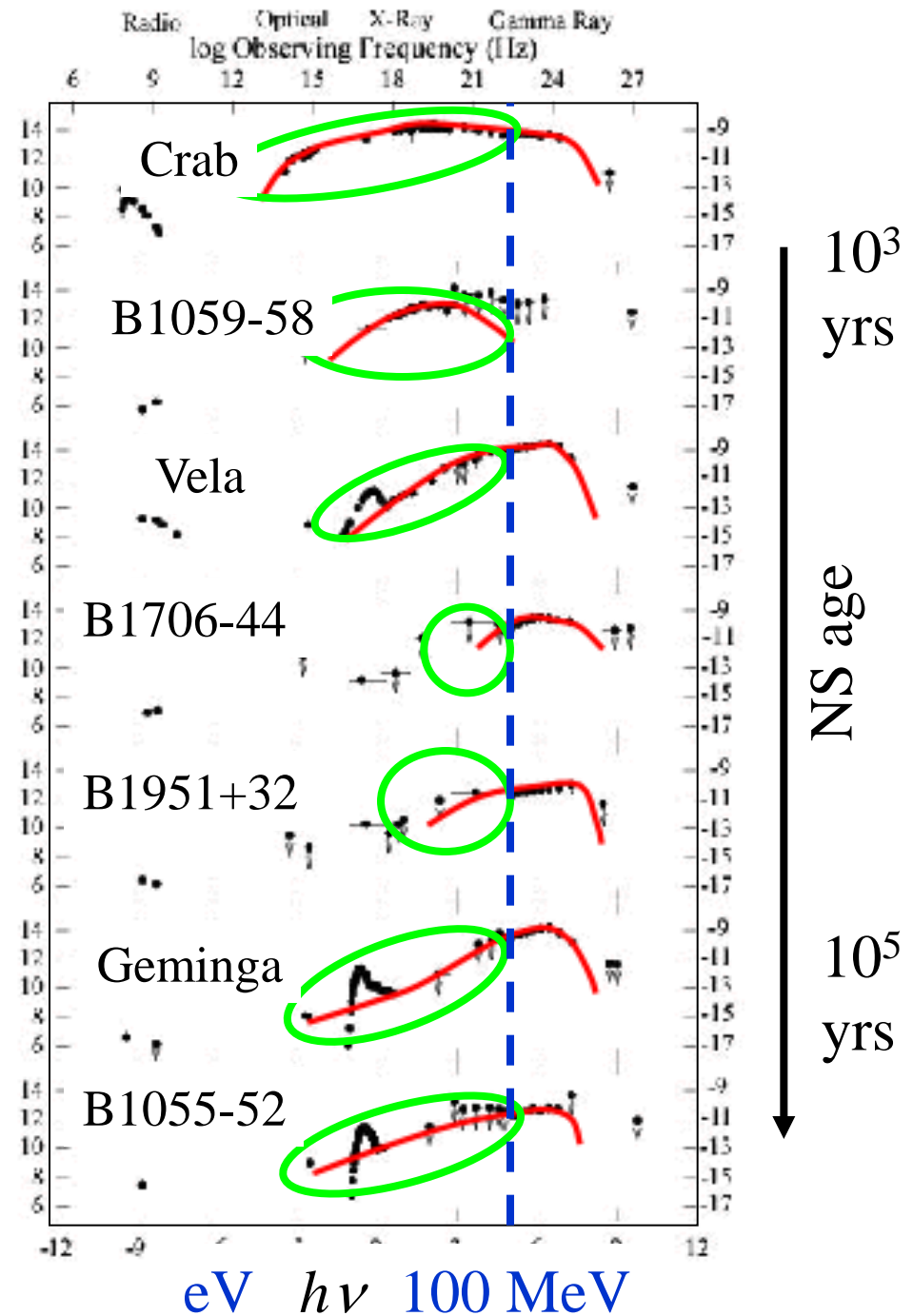
● Above 20 GeV , **ICS by secondary/tertiary pairs** contributes.



Broad-band spectra (pulsed)

- High-energy ($> 100\text{MeV}$) photons are emitted mainly via **curvature** process by ultra-relativistic ($\sim 10\text{TeV}$) e^\pm 's accelerated in pulsar magnetosphere.

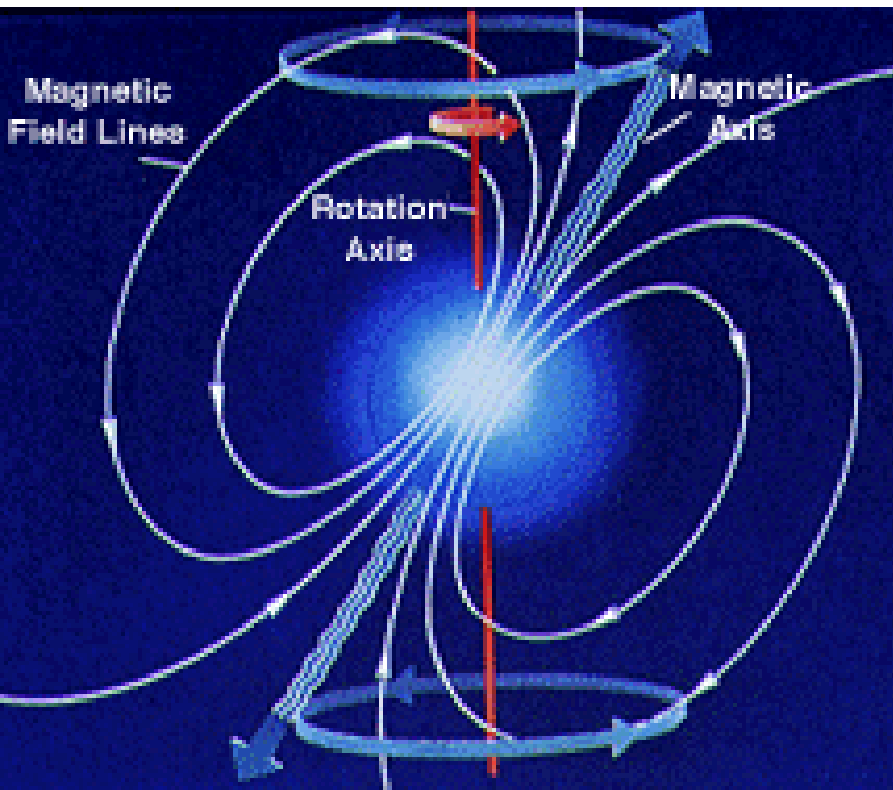
- Some of the **primary γ -rays** are absorbed in the NS magnetosphere and **reprocessed** in lower energies via **synchrotron** process.



§2 Rotating NS Magnetosphere

The observed high-energy emissions are realized when the rotational energy of the NS is electro-dynamically extracted and partly dissipated in its magnetosphere.

(e.g., unipolar inductor)

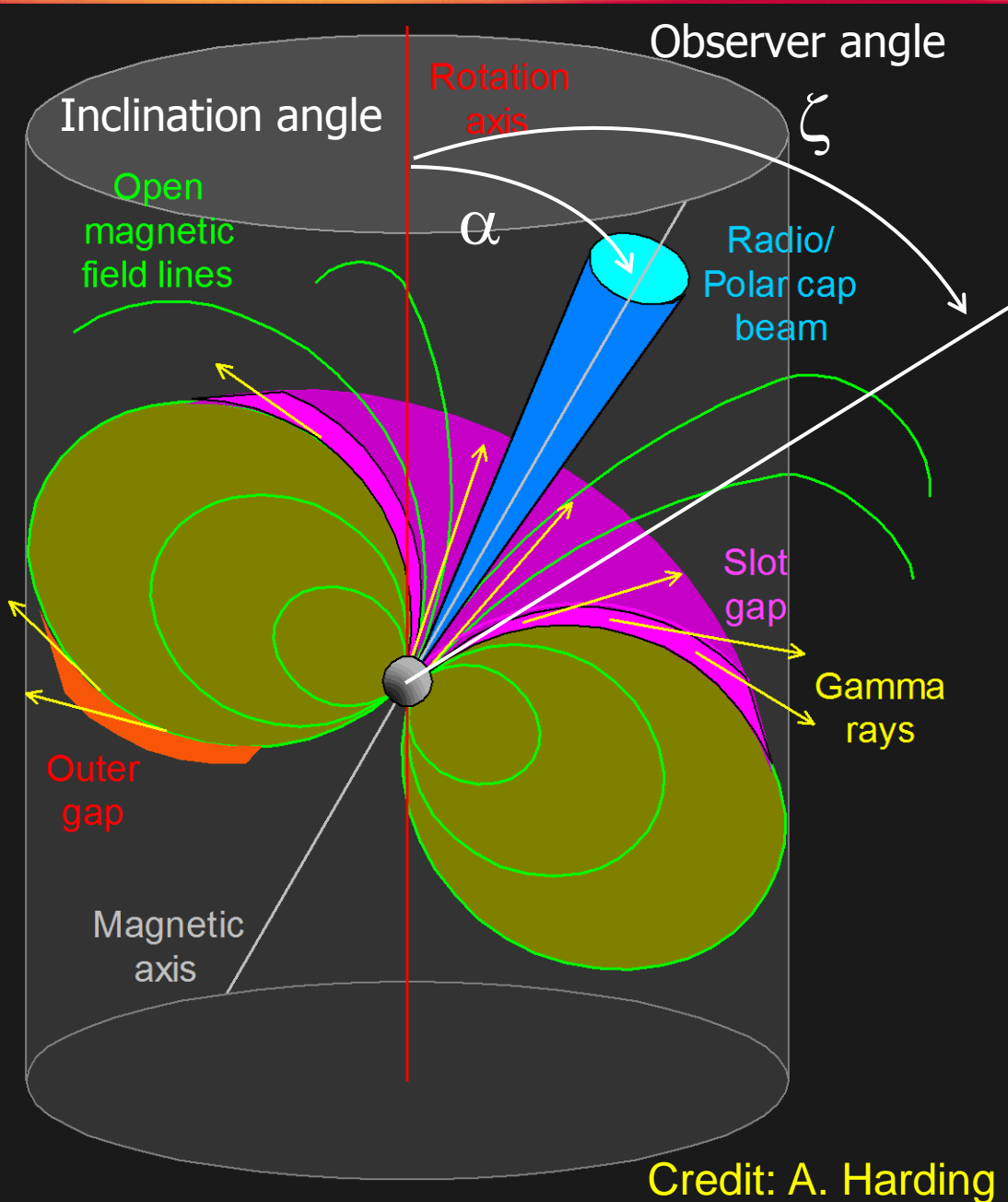


Magnetic and rotation axes are generally misaligned.

Pulsars:

rapidly rotating, highly magnetized NS

§2 Rotating NS Magnetosphere



Possible sites of particle acceleration

- Ideal MHD condition holds in most of magnetosphere,
 $E \cdot B = 0$.
- In some limited regions, deficient charge supply leads to $E \cdot B \neq 0$.
- In charge deficit region, E_{\parallel} is solved from the Poisson eq.,

$$\nabla \cdot E_{\parallel} = 4\pi(\rho - \rho_{GJ}).$$

Credit: A. Harding

§2 Rotating NS Magnetosphere

Early 80's, the **polar-cap (PC) model** was proposed.

(Daugherty & Harding ApJ 252, 337, 1982)

A single PC beam can produce a variety of pulse profiles.

However, the emission solid angle ($\Delta\Omega \ll 1$ ster) was too small to reproduce the wide-separated double peaks.

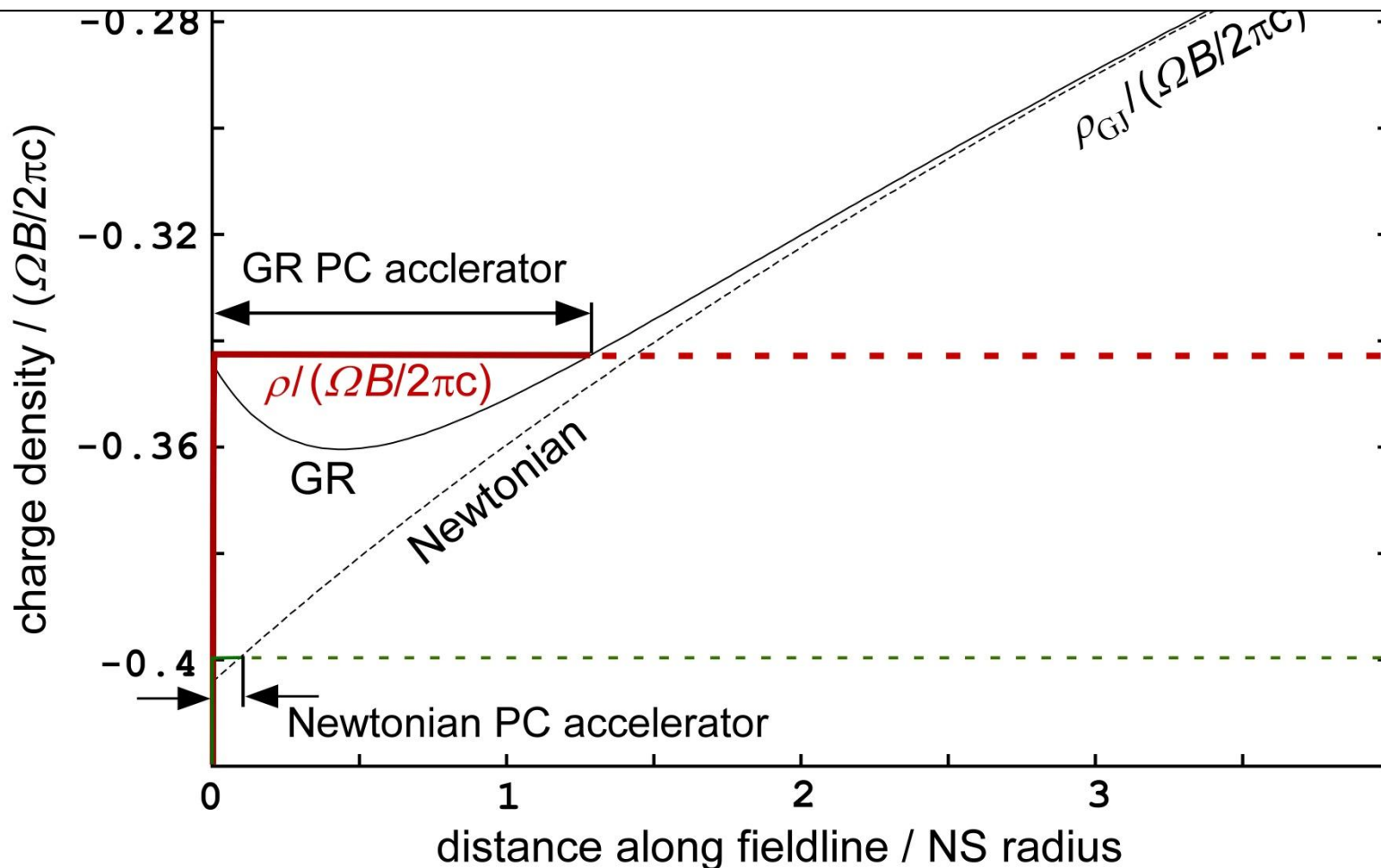
A great deal of effort has been made; however, one has to invoke a **very small inclination, α , and viewing angles, ζ** , to reproduce the widely separated pulse peaks.

In addition, localization of gap altitudes ($\ll r_*$) prohibits enough L_γ ($< 0.3L_{\text{spin}}$) as observed. ($L_{\text{radio}} \sim 10^{-5}L_{\text{spin}}$ is OK.)

Thus, a **high-altitude emission** drew attention.

§3 Higher-altitude Pulsar Emission Models

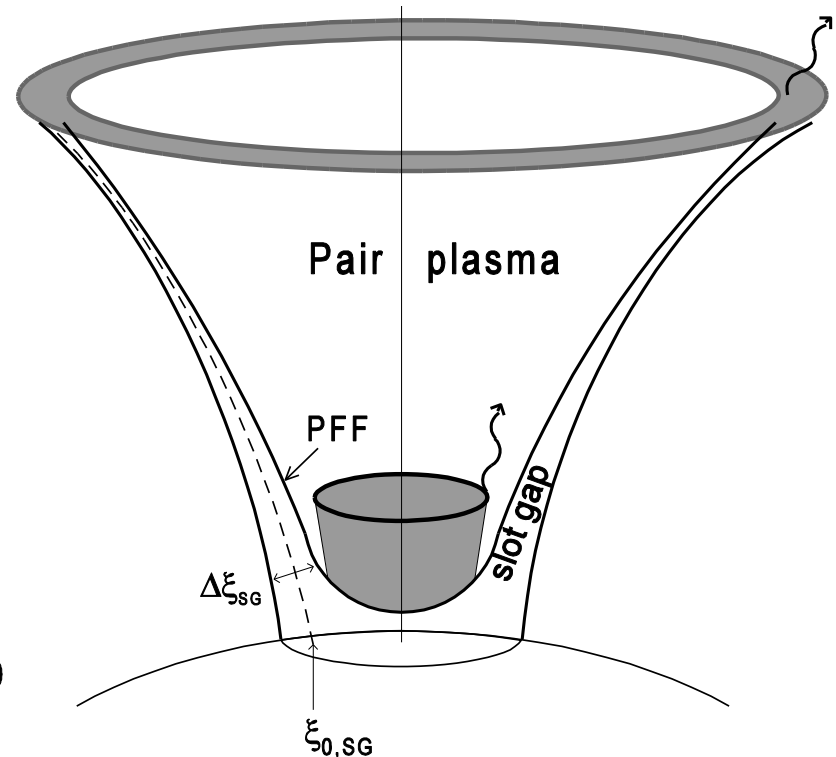
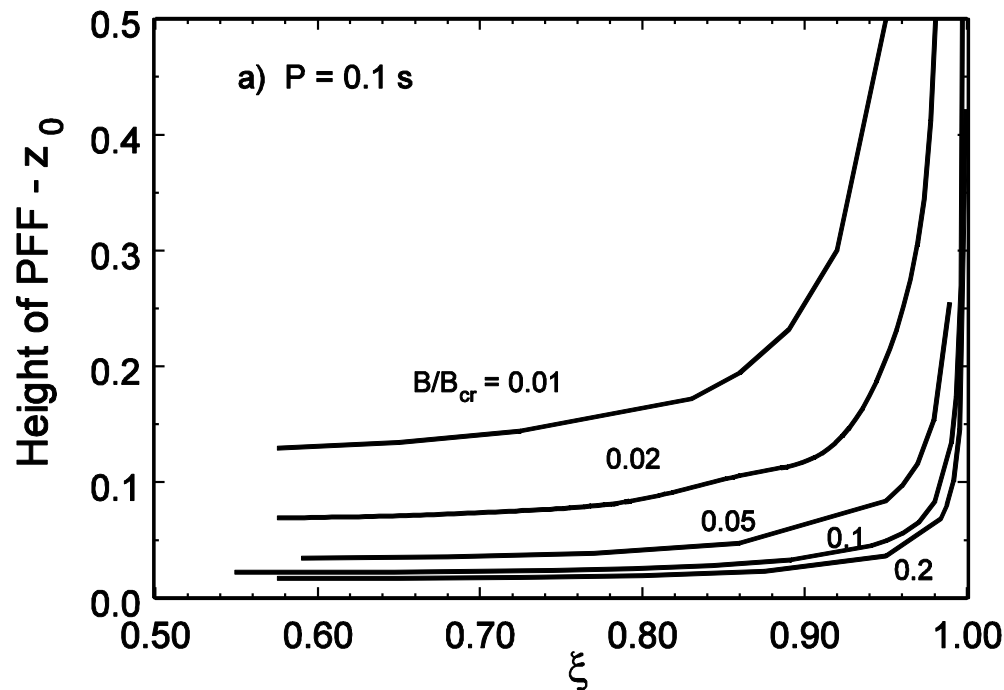
Big breakthrough: [Muslimov & Tsygan \(1992, MN 255, 61\)](#) found that a polar gap can extend into the higher altitudes by virtue of the **frame-dragging effect**.



§3 Higher-altitude Pulsar Emission Models

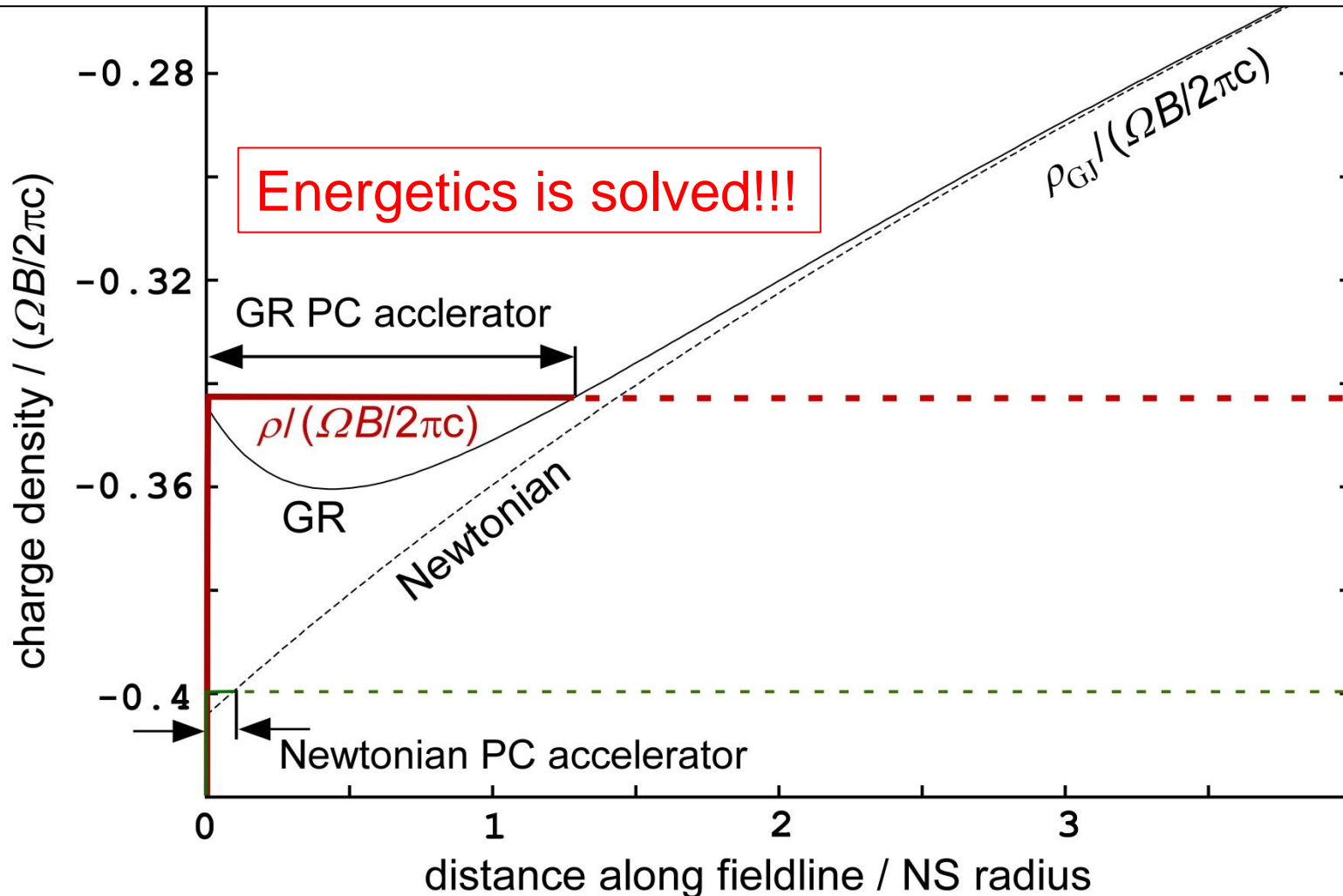
Extending the original idea of [Arons \(1983, ApJ 266, 215\)](#), [Muslimov & Harding \(2003, ApJ 588, 430\)](#) proposed the **lower-altitude slot-gap (SG) model**.

Slot gap = a pair-free space formed between the last-open field lines and the pair-formation front (PFF).



§3 Higher-altitude Pulsar Emission Models

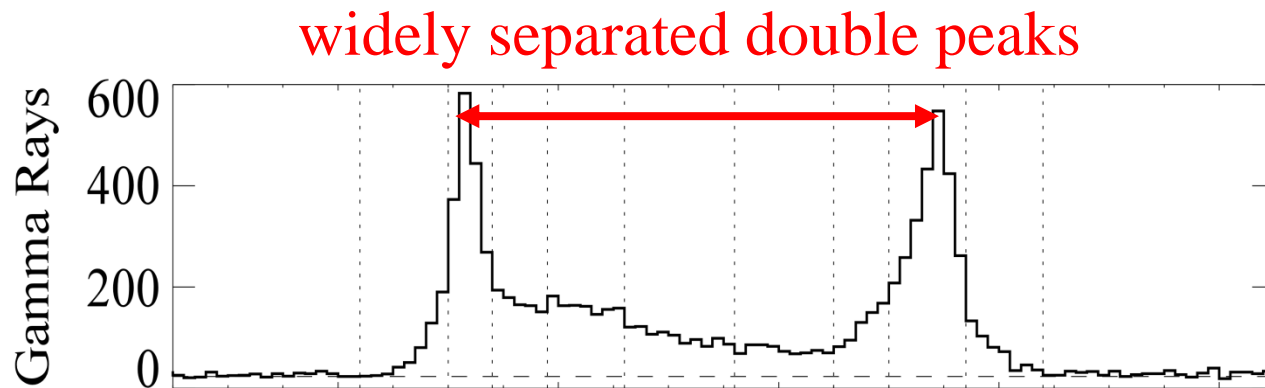
However, a **lower-altitude SG** is still limited within a few r_* .
Thus, the same difficulty ($\Delta\Omega \ll 1$ ster) still remains.



§3 Higher-altitude Pulsar Emission Models

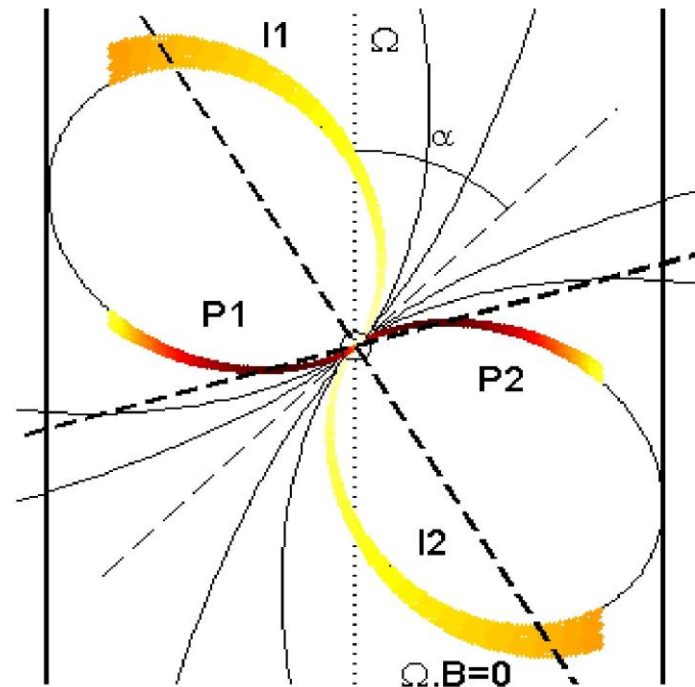
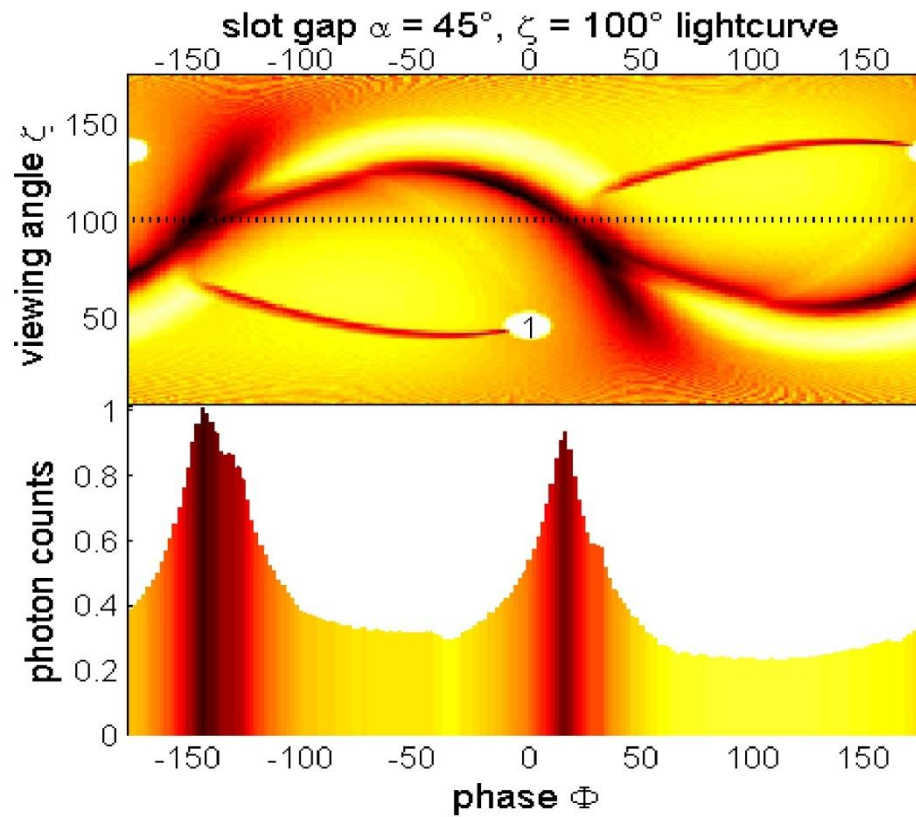
To contrive a higher-altitude emission model,
Muslimov & Harding (2004a, ApJ 606, 1143; 2004b, ApJ 617, 471) and **Dyks, Harding & Rudak (2004, ApJ 606, 1125)**
extended the lower-altitude slot-gap solution into higher
altitudes (in fact, by hand).

They explained, e.g., the widely separated double peaks.



§3 Higher-altitude Pulsar Emission Models

Assuming that the **gap extends** from the NS surface **to the light cylinder** with constant emissivity, **Dyks & Rudak (2003, ApJ 598, 1201)** demonstrated the formation of double peaks, which arise from the crossing of two caustics associated with different poles.



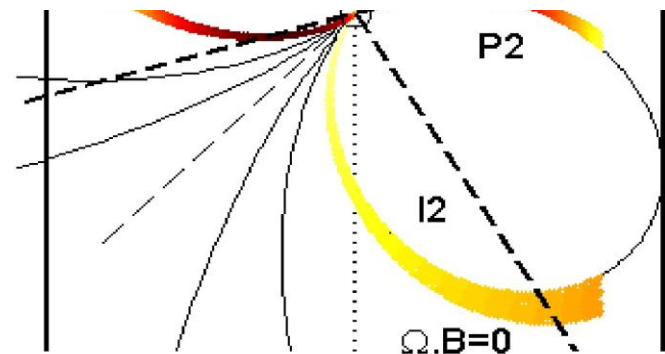
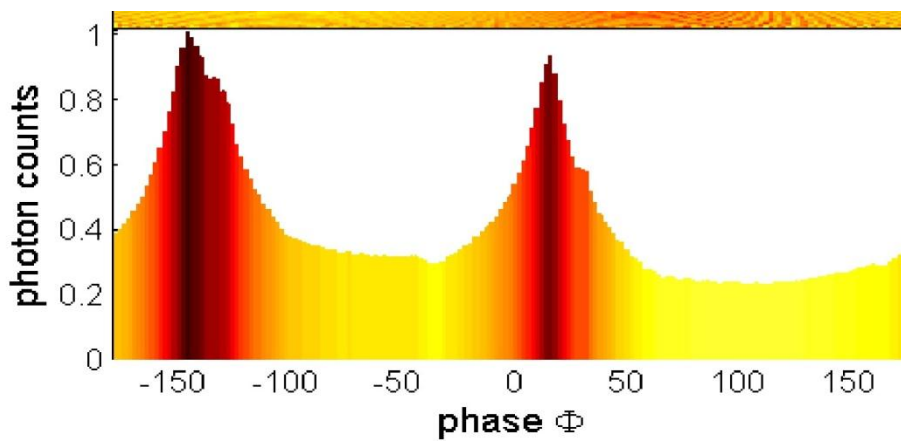
§3 Higher-altitude Pulsar Emission Models

Assuming that the **gap extends** from the NS surface **to the light cylinder** with constant emissivity, **Dyks & Rudak** (2003, ApJ 598, 1201) demonstrated the formation of double peaks.

In this **higher-altitude slot-gap model**, most observers catch emission from both (N/S) poles.

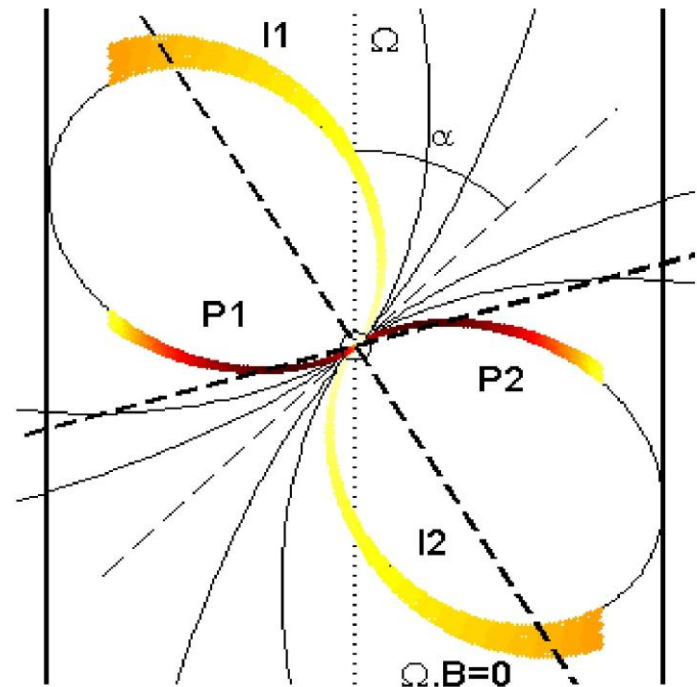
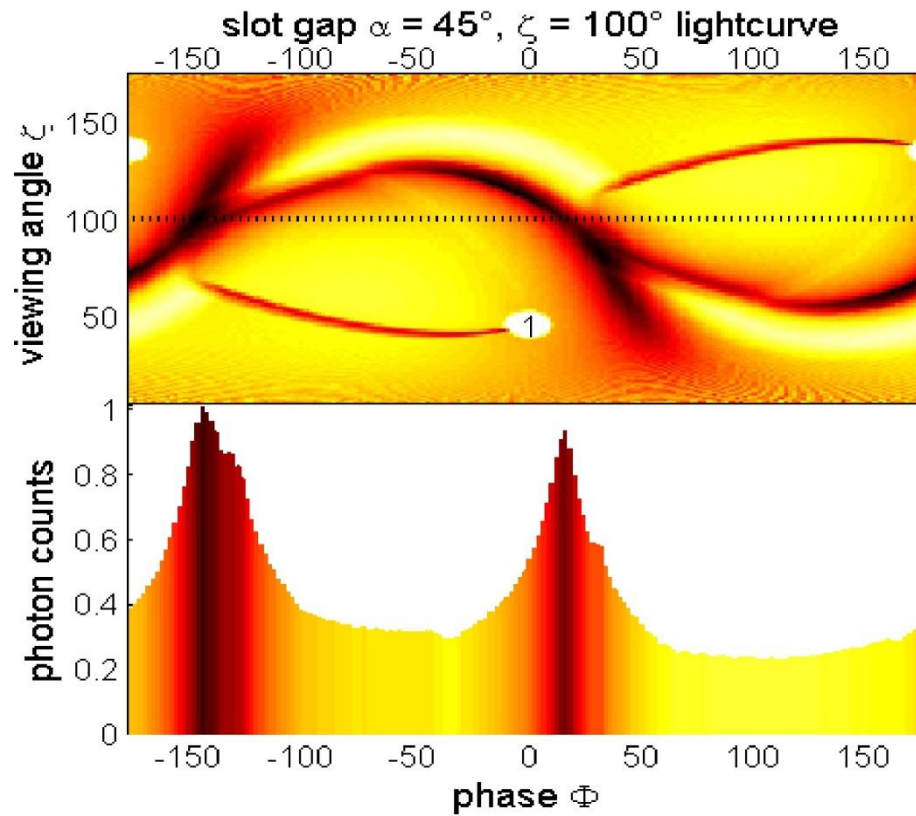
trailing-side emission \rightarrow main peaks

leading-side emission \rightarrow inter-peak & off-pulse



§3 Higher-altitude Pulsar Emission Models

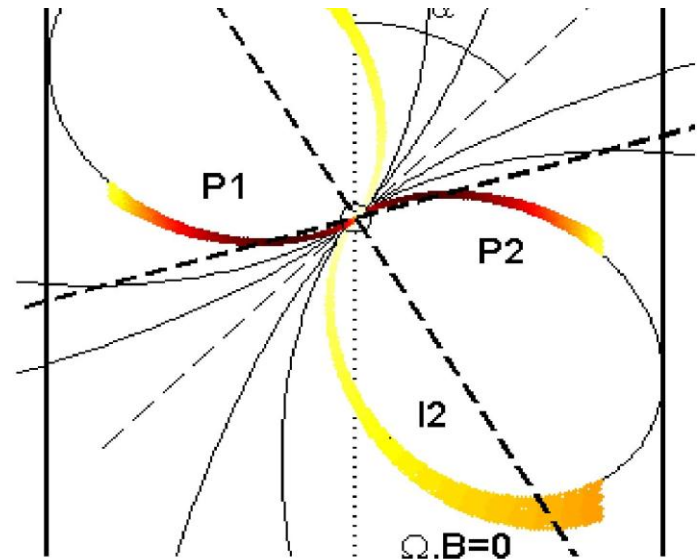
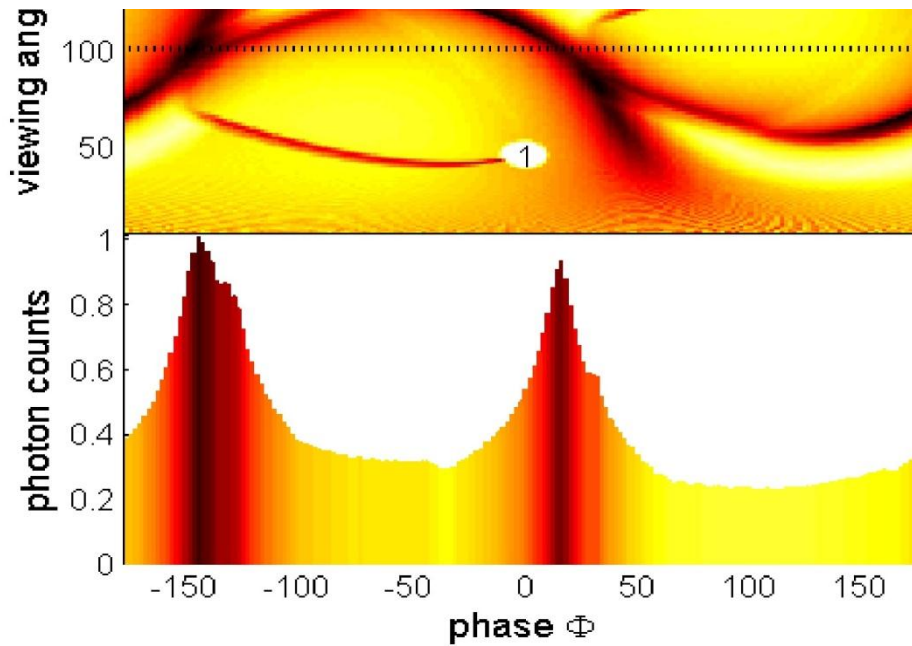
Dyks, Harding, Rudak (2004, ApJ 606, 1125) showed that Crab pulsar's optical polarization characteristics can be qualitatively reproduced by their SG model.



§3 Higher-altitude Pulsar Emission Models

Dyks, Harding, Rudak (2004, ApJ 606, 1125) showed that Crab pulsar's optical polarization characteristics can be qualitatively reproduced by their SG model.

Harding+ (2008, ApJ 680, 1378) examined Crab's γ -ray emission and found that the pulse profiles and the phase-resolved spectrum are qualitatively reproduced.

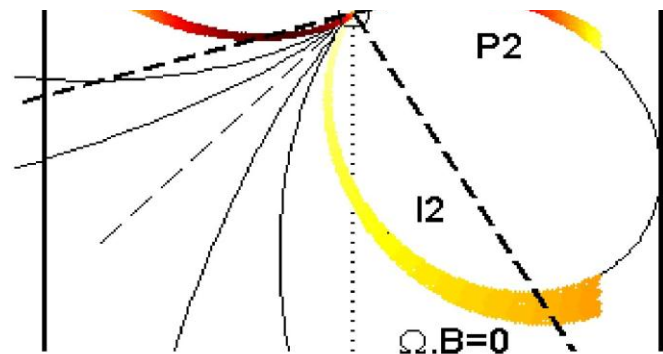
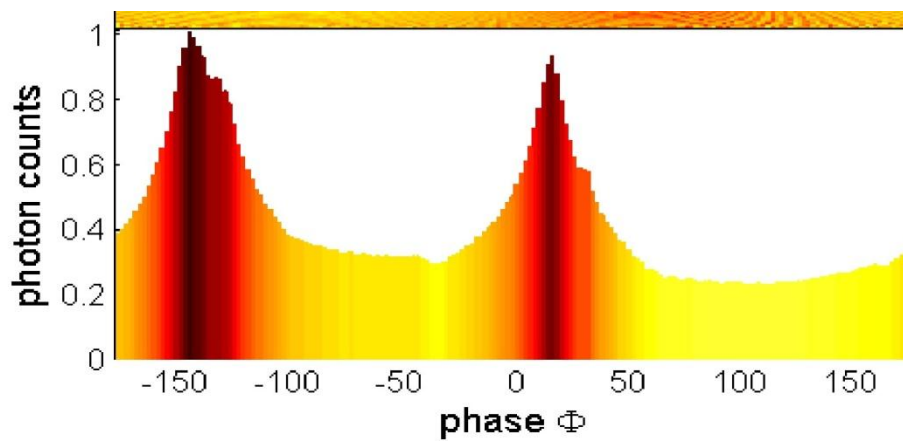


§3 Higher-altitude Pulsar Emission Models

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Harding+ (2008, ApJ 680, 1378) examined Crab's γ -ray emission and found that the pulse profiles and the phase-resolved spectrum are qualitatively reproduced.

However, unfortunately, the higher-altitude SG model contains two fatal electro-dynamical inconsistencies.



§4 Difficulties in SG Model: Insufficient Luminosity

Problem 1: insufficient luminosity

KH (2008) ApJ 688, L25

Adopting the same parameter as Harding+(2008), one obtains too small γ -ray flux from a higher-altitude SG.

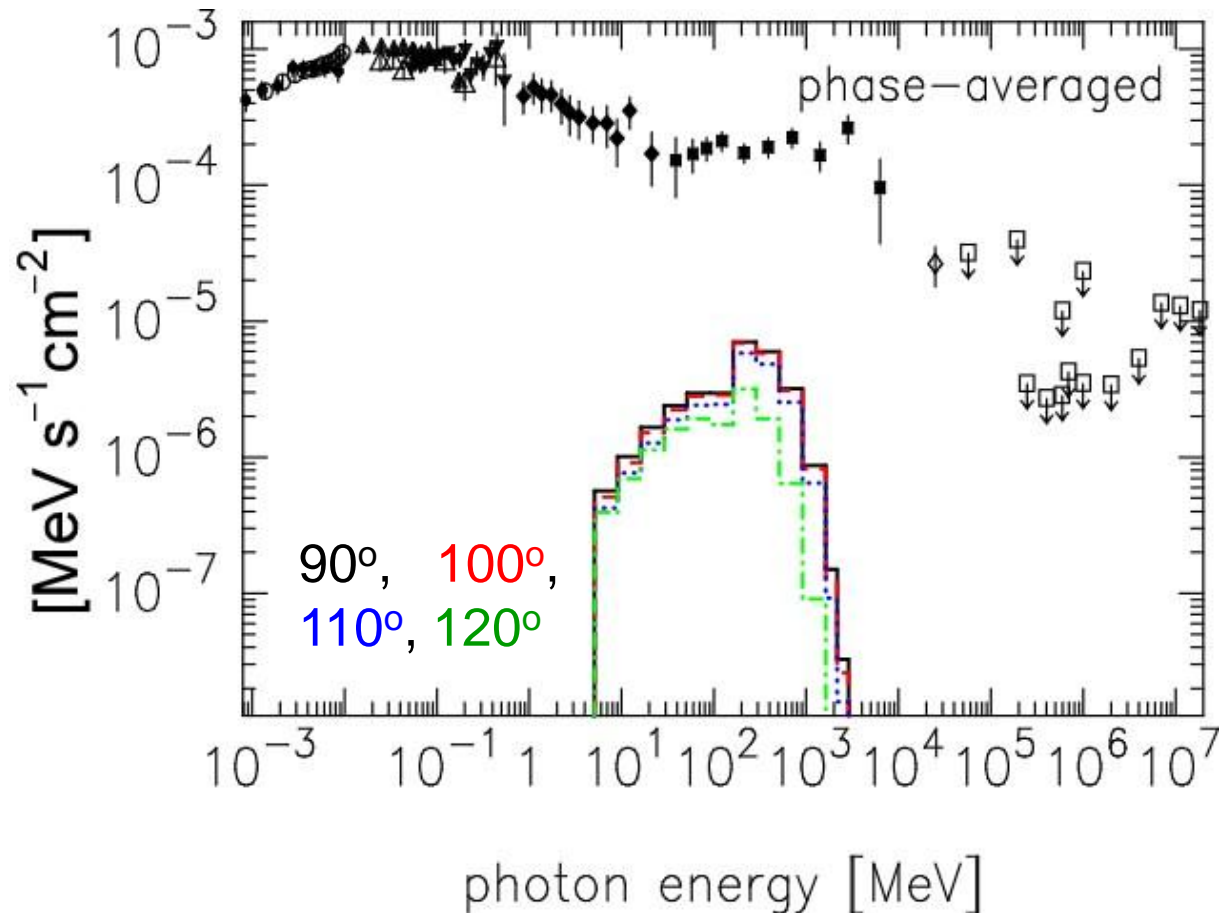


Fig. Phase-averaged SG spectrum for four discrete viewing angles, 90° , 100° , 110° , and 120° .

§4 Difficulties in SG Model: Insufficient Luminosity

Problem 1: insufficient luminosity

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Adopting the same parameter as Harding+(2008), one obtains too small γ -ray flux from a higher-altitude SG.

Analytically predicted γ -ray flux of the Crab pulsar:

$$(\nu F_\nu)_{\text{peak}} \approx 0.0450 f^3 \kappa \underbrace{\frac{\mu^2 \Omega^4}{c^3} \frac{1}{d^2}}_{\propto \dot{E}/d^2 : \text{spin-down flux}}, \quad \kappa \sim 1.$$

f : fractional gap width ($f \ll 1$ denotes a thin gap)

The difference between OG and SG models appears through f , κ , and assumed μ (magnetic moment).

§4 Difficulties in SG Model: Insufficient Luminosity

Problem 1: insufficient luminosity

KH (2008) ApJ 688, L25

$$(\nu F_\nu)_{\text{peak}} \approx 0.0450 f^3 \kappa \frac{\mu^2 \Omega^4}{c^3} \frac{1}{d^2}$$

Apply this general result to the Crab pulsar ($\Omega=190 \text{ rad s}^{-1}$).

(I) For **OG** model ($f \sim 0.14$, $\kappa \sim 0.3$, $\mu = 4 \times 10^{30} \text{ G cm}^3$),

$$(\nu F_\nu)_{\text{peak}} \sim 4 \times 10^{-4} \text{ MeV s}^{-1} \text{ cm}^{-2} \sim \text{EGRET flux.}$$

(II) For **SG** model ($f \sim 0.04$, $\kappa \sim 0.2$), even with a large μ ,

$$\begin{aligned} (\nu F_\nu)_{\text{peak}} &\sim 3 \times 10^{-5} (\mu / 8 \times 10^{30})^2 \text{ MeV s}^{-1} \text{ cm}^{-2} \\ &< 0.1 \text{ EGRET flux.} \end{aligned}$$

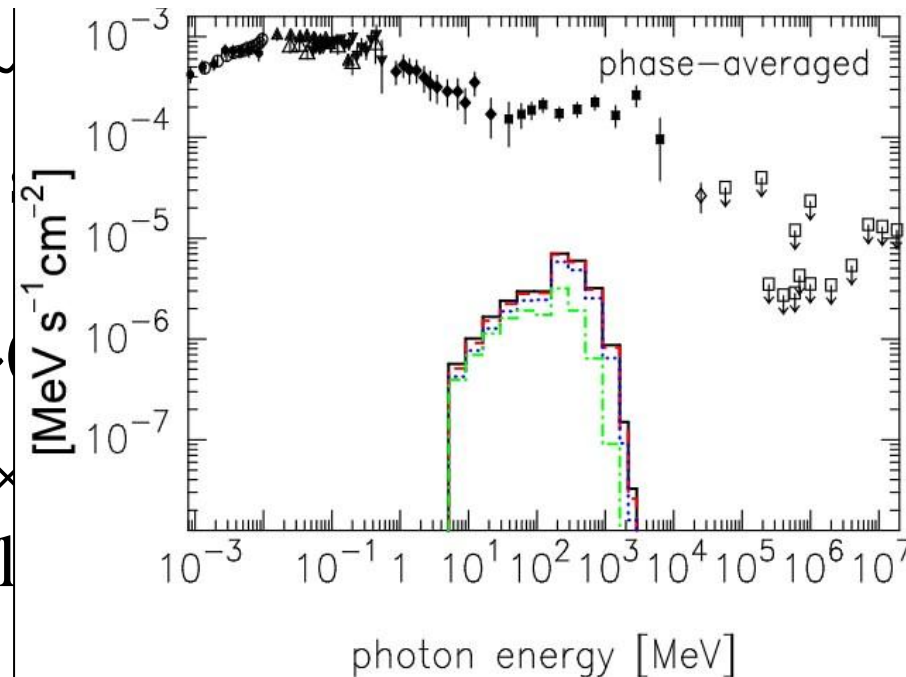
§4 Difficulties in SG Model: Insufficient Luminosity

Problem 1: insufficient luminosity

In short, both analytical and numerical results show that **a SG can produce a negligible γ -ray flux.**

(I) For **OG** model ($f \sim 0.14$, $\kappa \sim$
 $(\nu F_\nu)_{\text{peak}} \sim 4 \times 10^{-4} \text{ MeV}$

(II) For **SG** model ($f \sim 0.04$, $\kappa \sim$
 $(\nu F_\nu)_{\text{peak}} \sim 3 \times 10^{-5} (\mu/8 \times$
 $< 0.1 \text{ EGRET fl}$

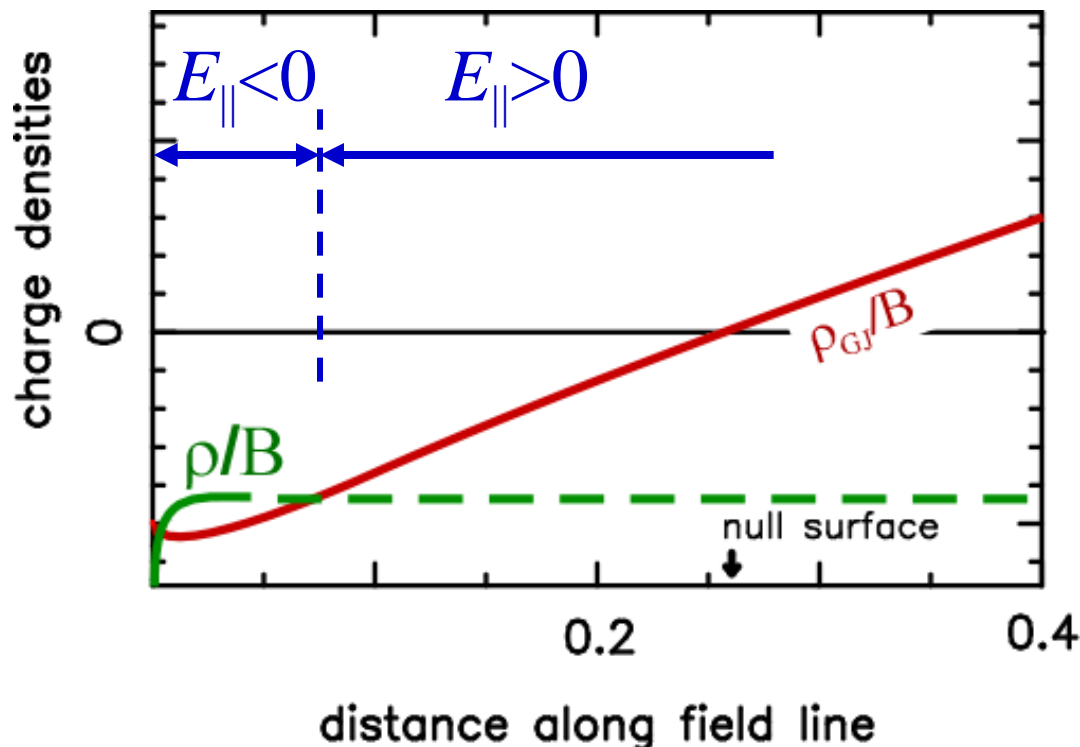


OG model remains as the only possible γ -ray pulsar model.

§5 Difficulties in SG Model: Unphysical GJ Charge

Problem 2: unphysical assumption of GJ charge density (per B flux tube)

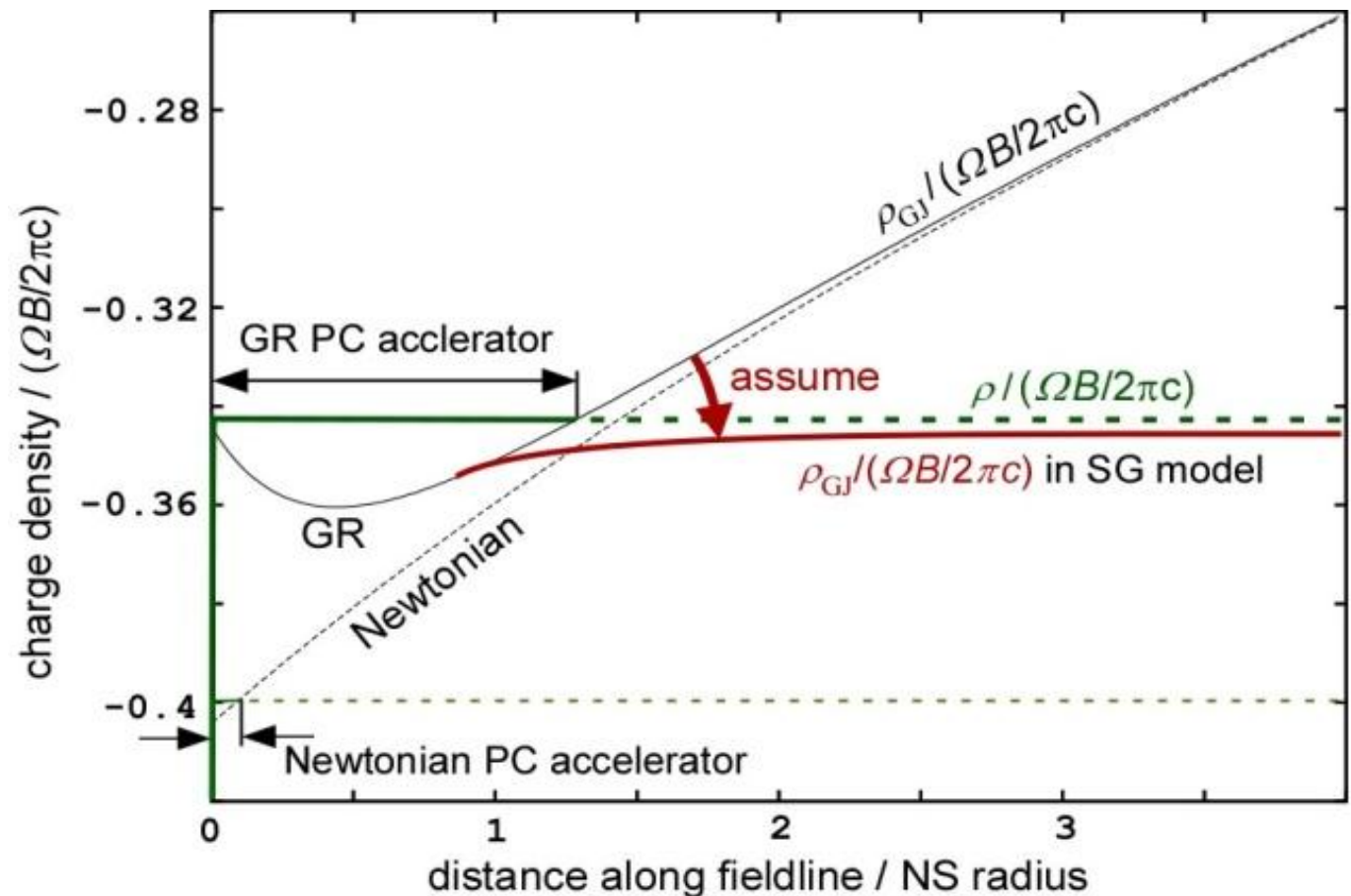
Without pair creation, electron density per B will be constant along the field line. However, it results in a reversal of E_{\parallel} due to the sign change of $\rho - \rho_{\text{GJ}}$.



§5 Difficulties in SG Model: Unphysical GJ Charge

Problem 2: unphysical assumption of ρ_{GJ}/B

To prevent a sign reversal of E_{\parallel} , they assumed that ρ_{GJ}/B tends to a constant in the higher altitudes



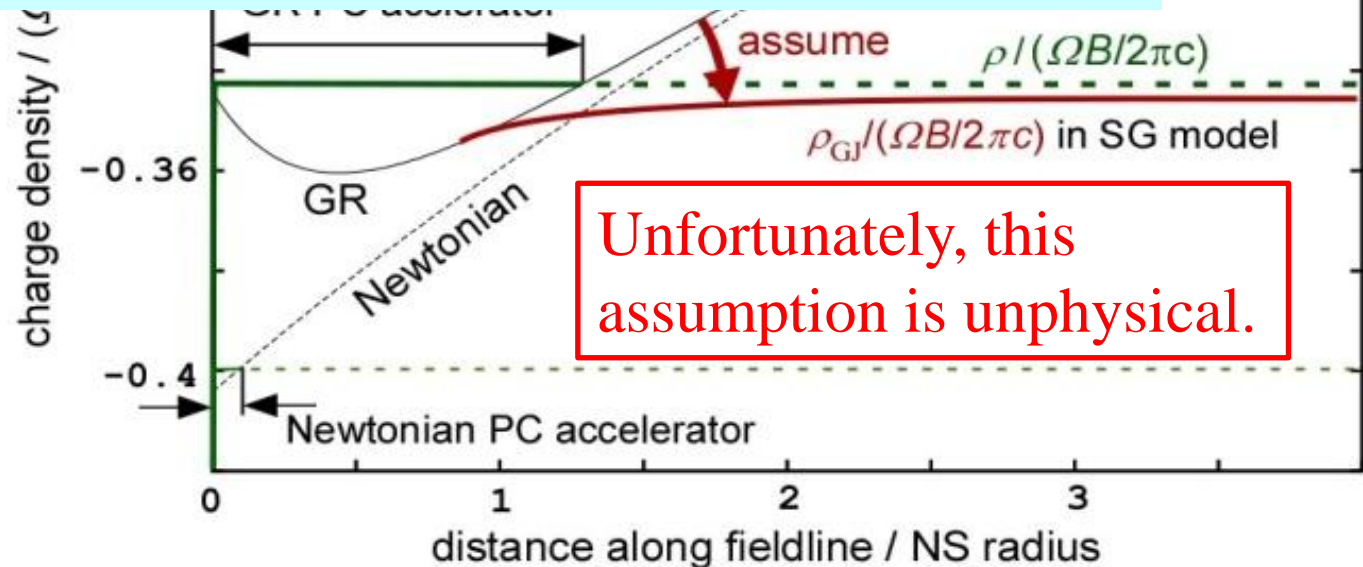
§5 Difficulties in SG Model: Unphysical GJ Charge

Problem 2: unphysical assumption of ρ_{GJ}/B

To prevent a sign reversal of E_{\parallel} , they assumed that ρ_{GJ}/B tends to a constant in the higher altitudes.

However, Maxwell eq. uniquely gives

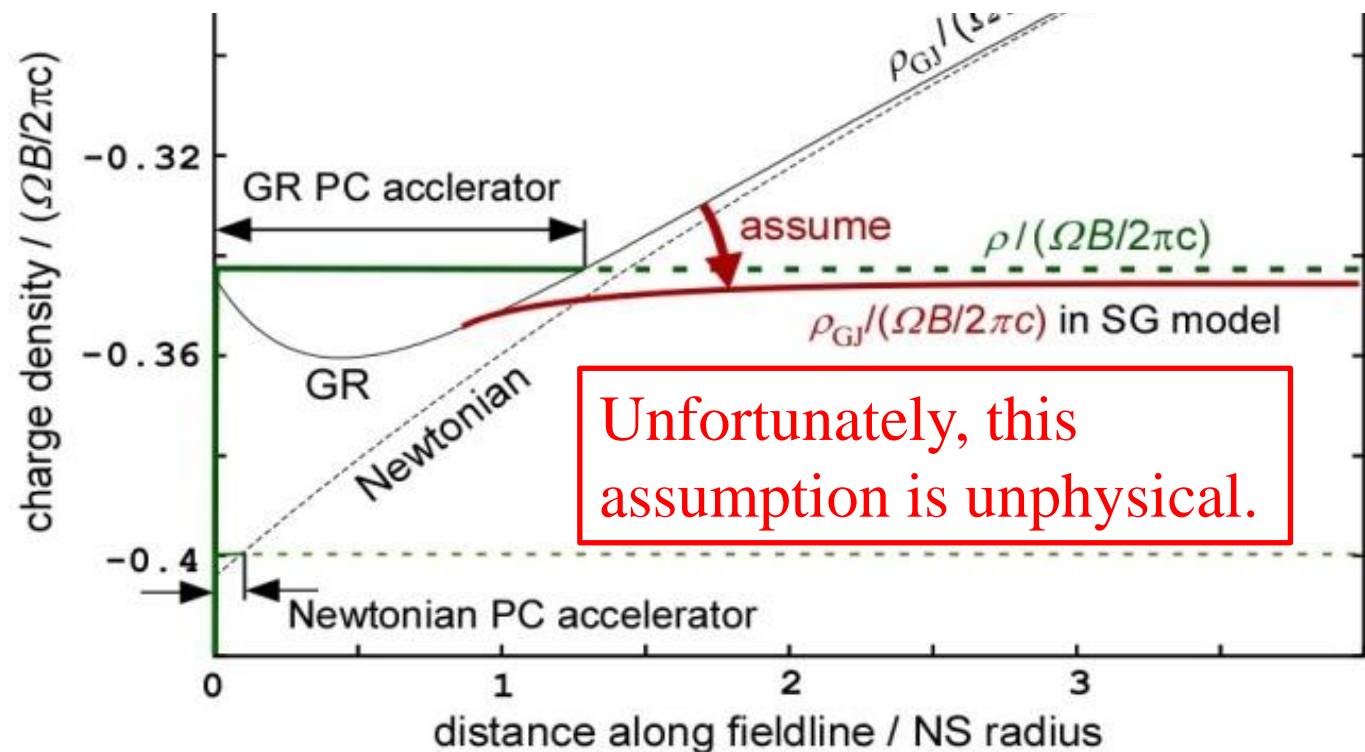
$$\rho_{\text{GJ}} = \frac{c^2}{4\pi\sqrt{-g}} \partial_{\mu} \left(\frac{\sqrt{-g}}{\rho_w^2} g^{\mu\nu} g_{\phi\phi} (\Omega - \omega) F_{\phi\nu} \right).$$



§5 Difficulties in SG Model: Unphysical GJ Charge

Problem 2: unphysical assumption of ρ_{GJ}/B

Since the pair-starved PC (PSPC) model adopts the same ρ_{GJ} distribution, this difficulty applies not only to the higher-altitude SG model, but also to the PSPC model (Venter+ 2009, ApJ 707, 800).

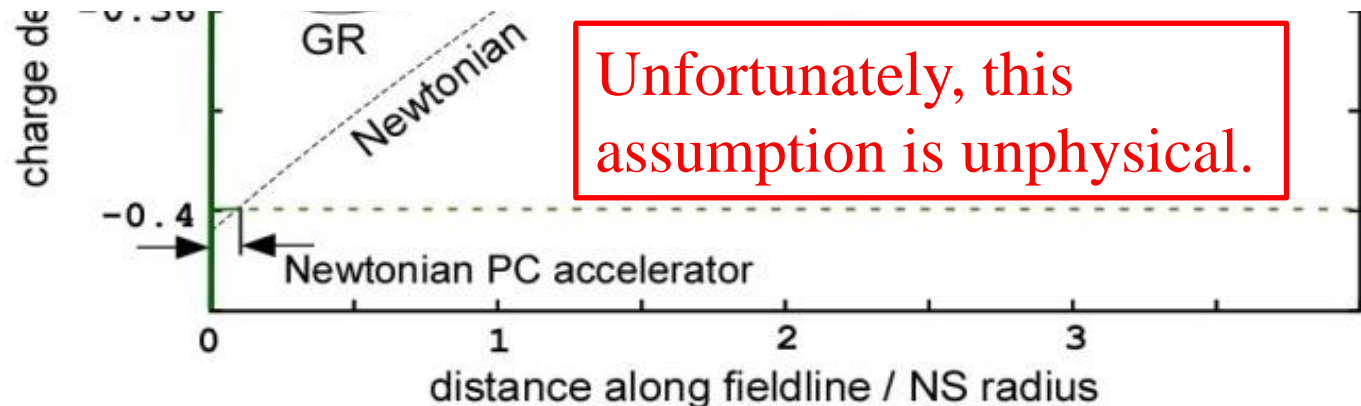


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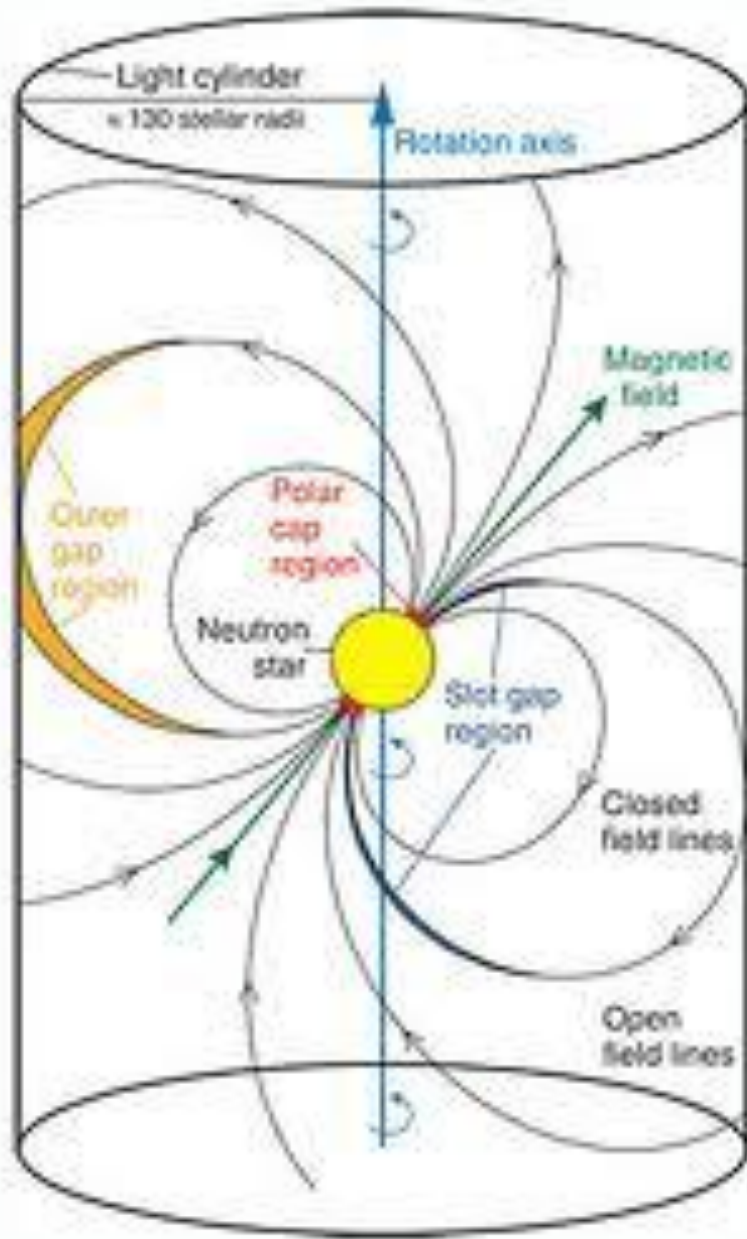
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In short, higher-latitude SG model & PSPC model contain serious electro-dynamical problem that contradicts with Maxwell eq.



§6 Classic Outer-gap (OG) Models



As an alternative possibility of high-altitude emission model, the **outer gap model** was proposed.

Cheng, Ho, Ruderman
(1986, ApJ 300, 500)

So far, there have been found no serious electrodynamical problems in the OG model (unlike SG or PSPC model).

Thus, let us concentrate on the OG model in what follows.

§6 Classic OG Models

Mid 80's, the **outer-gap (OG) model** was proposed.

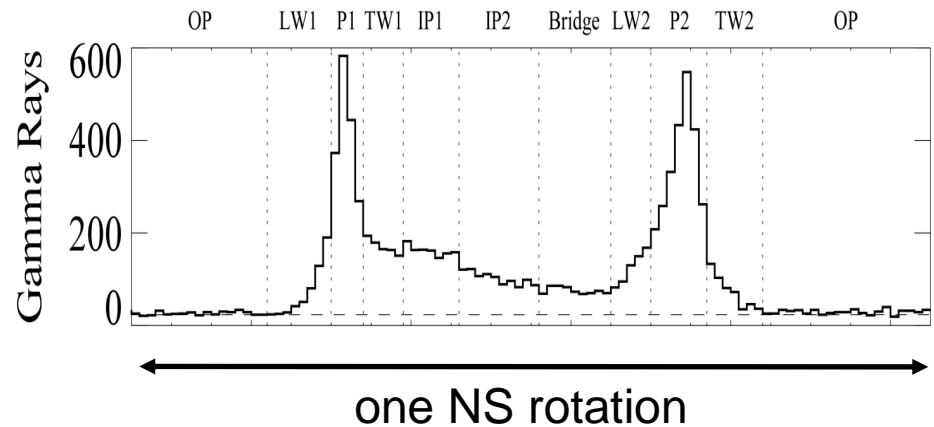
(Cheng, Ho, Ruderman ApJ 300, 500, 1986)

Emission altitude $> 100 r_{\text{NS}}$ \longrightarrow hollow cone emission
($\Delta\Omega > 1$ ster)

Mid 90s', OG model was further developed by including
special relativistic effects. (Romani ApJ 470, 469)

\longrightarrow Explains wide-separated double peaks.

Outer-gap model
became promising.



§6 *Classic OG Models*

Various attempts have been made on recent OG model:

3-D geometrical model

- phase-resolved spectra (Cheng + '00; Tang + '08)
- atlas of light curves for PC, OG, SG models
(Watters + '08)

2-D self-consistent solution (Takata + '06; KH '06)

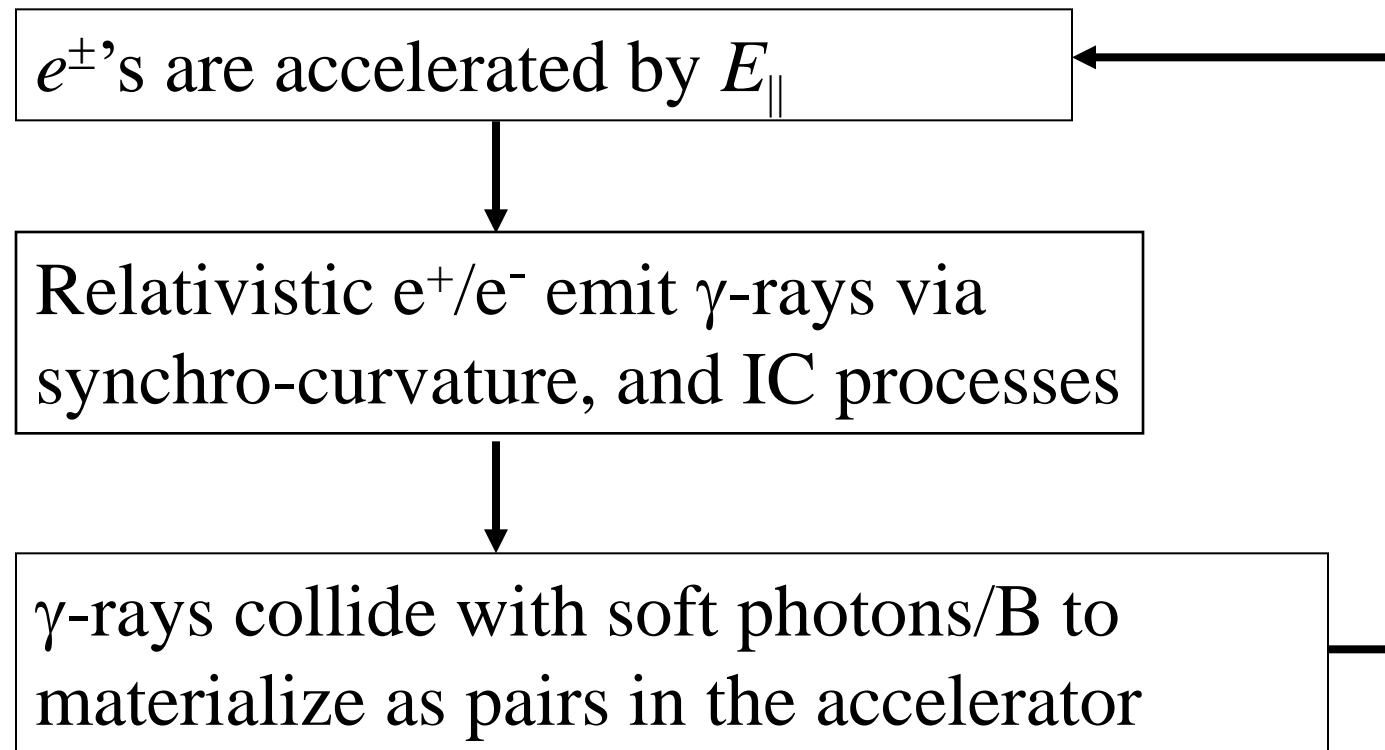
3-D self-consistent solution

- phase-resolved spectra, absolute luminosity
if we give only P , dP/dt , α , kT (+ ζ) (this talk)

In this talk, I'll present the most recent results obtained in my 3-D version of self-consistent OG calculations.

§7 *Modern Outer-gap Model*

Self-sustained pair-production cascade in a rotating NS magnetosphere:



§7 Modern Outer-gap Model

The **Poisson equation** for the electrostatic potential ψ is given by

$$-\nabla^2\psi = 4\pi(\rho - \rho_{\text{GJ}}) ,$$

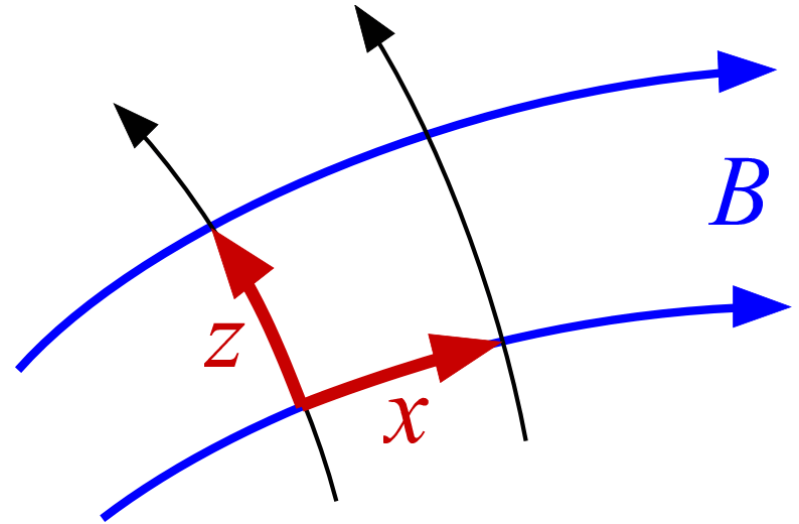
where

$$E_{\parallel} \equiv -\frac{\partial\Psi}{\partial x} , \quad \rho_{\text{GJ}} \equiv -\frac{\mathbf{\Omega}\cdot\mathbf{B}}{2\pi c} ,$$

$$\rho \equiv e \int_0^{\infty} d\mathbf{p}^3 \left[N_+(\mathbf{x}, \mathbf{p}) - N_-(\mathbf{x}, \mathbf{p}) \right] + \rho_{\text{ion}} .$$

N_+/N_- : distrib. func. of e^+/e^-

\mathbf{p} : momentum of e^+/e^-



§7 Modern Outer-gap Model

Assuming $\partial_t + \Omega \partial_\phi = 0$, we solve the e^\pm 's Boltzmann eqs.

$$\frac{\partial N_\pm}{\partial t} + \vec{v} \cdot \nabla N_\pm + \left(e \vec{E}_\parallel + \frac{\vec{v}}{c} \times \vec{B} \right) \cdot \frac{\partial N_\pm}{\partial \vec{p}} = S_{IC} + S_{SC} + \int \alpha_\nu d\nu \int \frac{I_\nu}{h\nu} d\omega$$

together with the radiative transfer equation,

$$\frac{dI_\nu}{dl} = -\alpha_\nu I_\nu + j_\nu$$

N_\pm : positronic/electronic spatial # density,

E_\parallel : magnetic-field-aligned electric field,

S_{IC} : ICS re-distribution function, $d\omega$: solid angle element,

I_ν : specific intensity, l : path length along the ray

α_ν : absorption coefficient, j_ν : emission coefficient

§7 *Modern Outer-gap Model*

Specify the three parameters: (period P is known)

- magnetic inclination (e.g., $\alpha_{\text{inc}}=45^\circ, 75^\circ$),
- magnetic dipole moment of NS (e.g., $\mu=4\times 10^{30}\text{G cm}^3$)
- neutron-star surface temperature (e.g., $kT_{\text{NS}}=50\text{ eV}$)

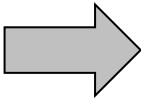
Solve Poisson eq. + Boltzmann eqs + radiative transf. eq.

I first solved (in **6-D** phase space)

- 3-D gap geometry,
 - acceleration electric field distribution, E_{\parallel} ,
 - particle density and energy spectrum,
 - photon specific intensity (\rightarrow predicts γ -ray properties),
- by specifying these three parameters, assuming \mathbf{B} -field structure by vacuum rotating dipole solution (Cheng + '00).

§8 *ICS spectrum of the Crab pulsar*

Let us apply this numerical method to the Crab pulsar.

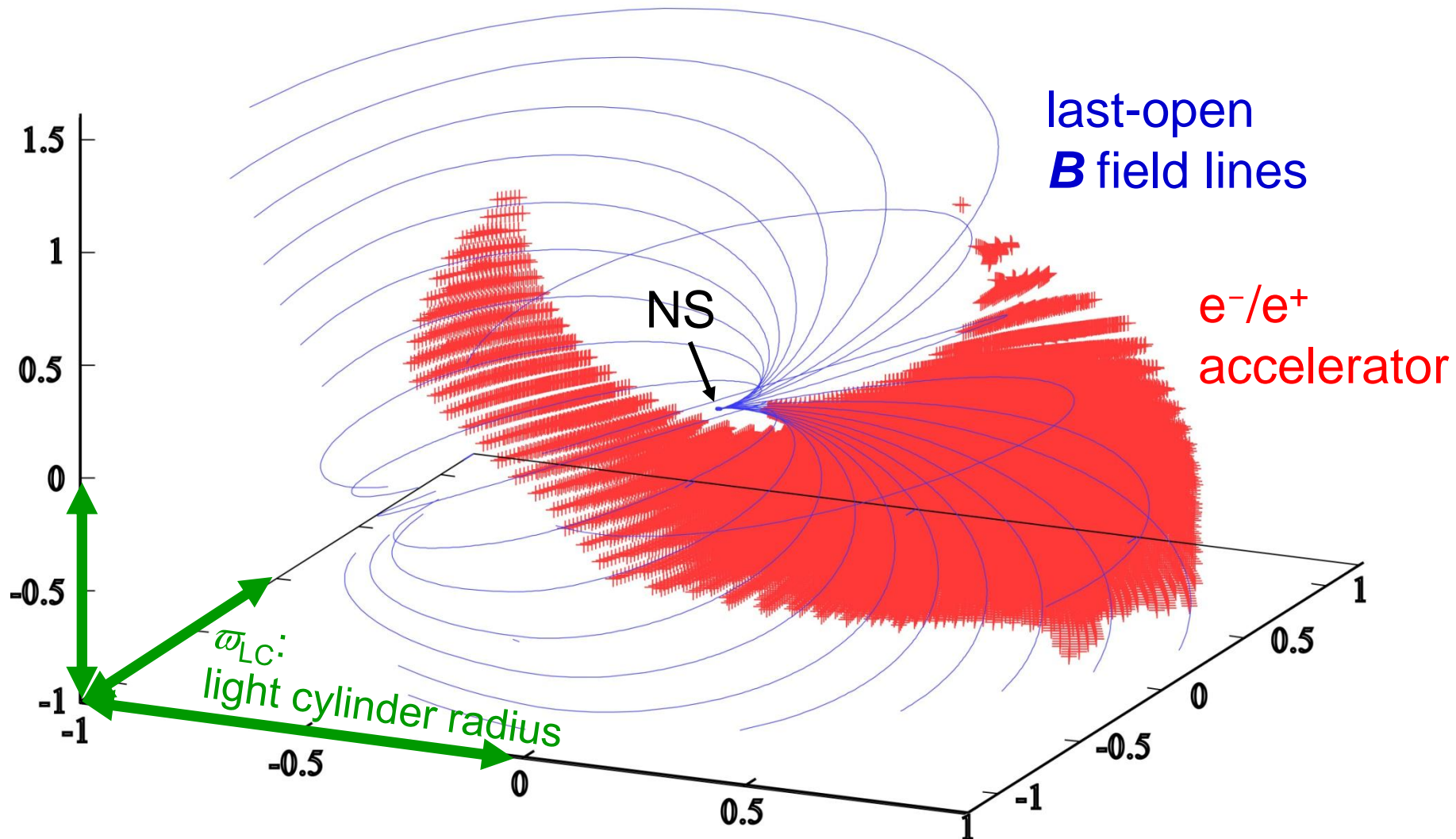
Maxwell &
Boltzmann eqs., 

- OG 3-D geometry,
- E_{\parallel} distribution,
- e^{+}/e^{-} distribution functions,
- photon specific intensity

Apply this method to the **Crab** pulsar, assuming
 $\mu = 3.8 \times 10^{30} \text{ G cm}^3$, $\alpha = 60^{\circ}$, $kT = 100 \text{ eV}$.

§8 *ICS spectrum of the Crab pulsar*

3-D distribution of the particle accelerator (i.e., high-energy emission zone) solved from the Poisson eq.:

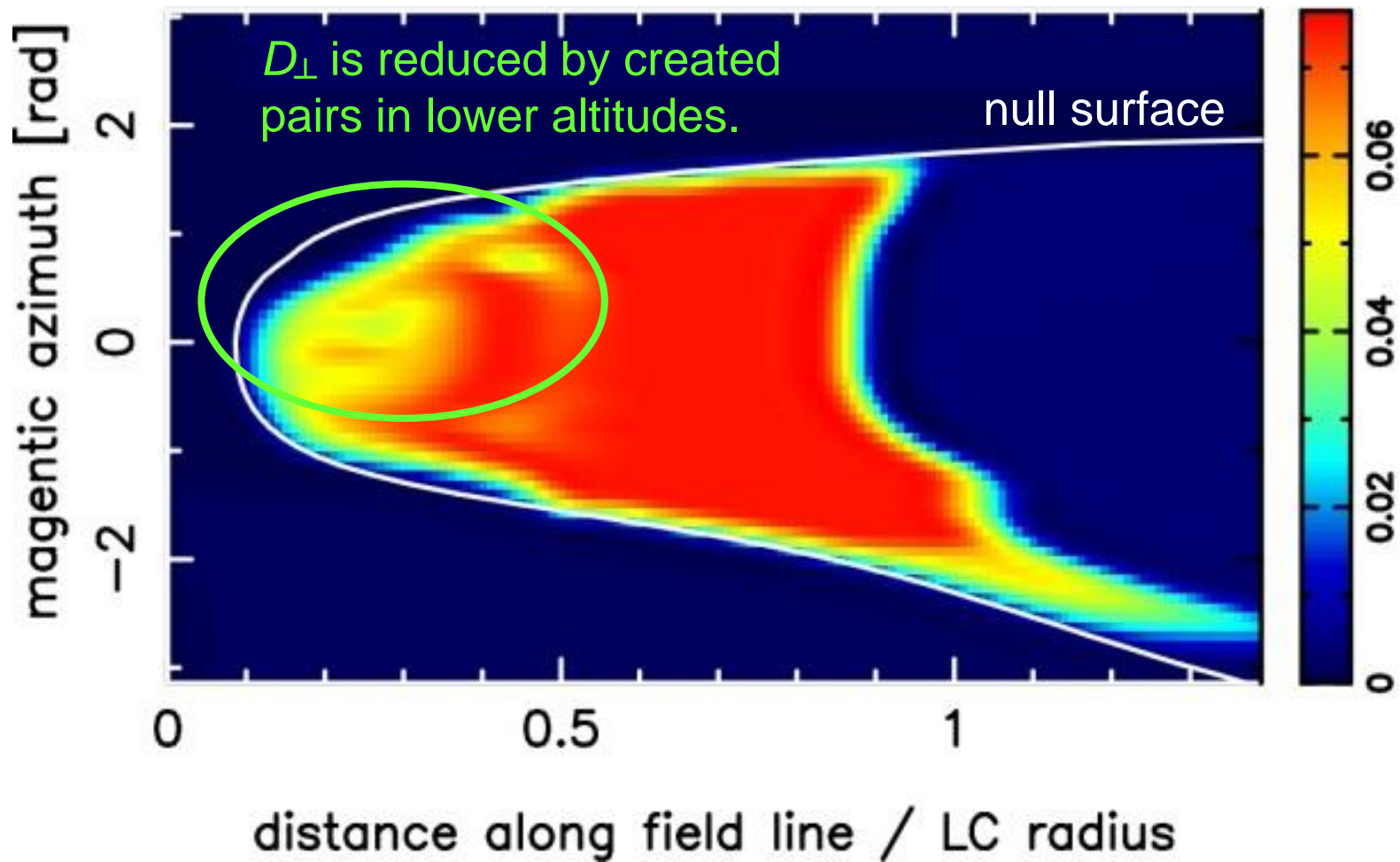


§8 HE/VHE Pulsation from the Crab Pulsar

3-D geometry: Trans-field gap thickness is self-regulated by pair production.

Crab, $\alpha=60^\circ$

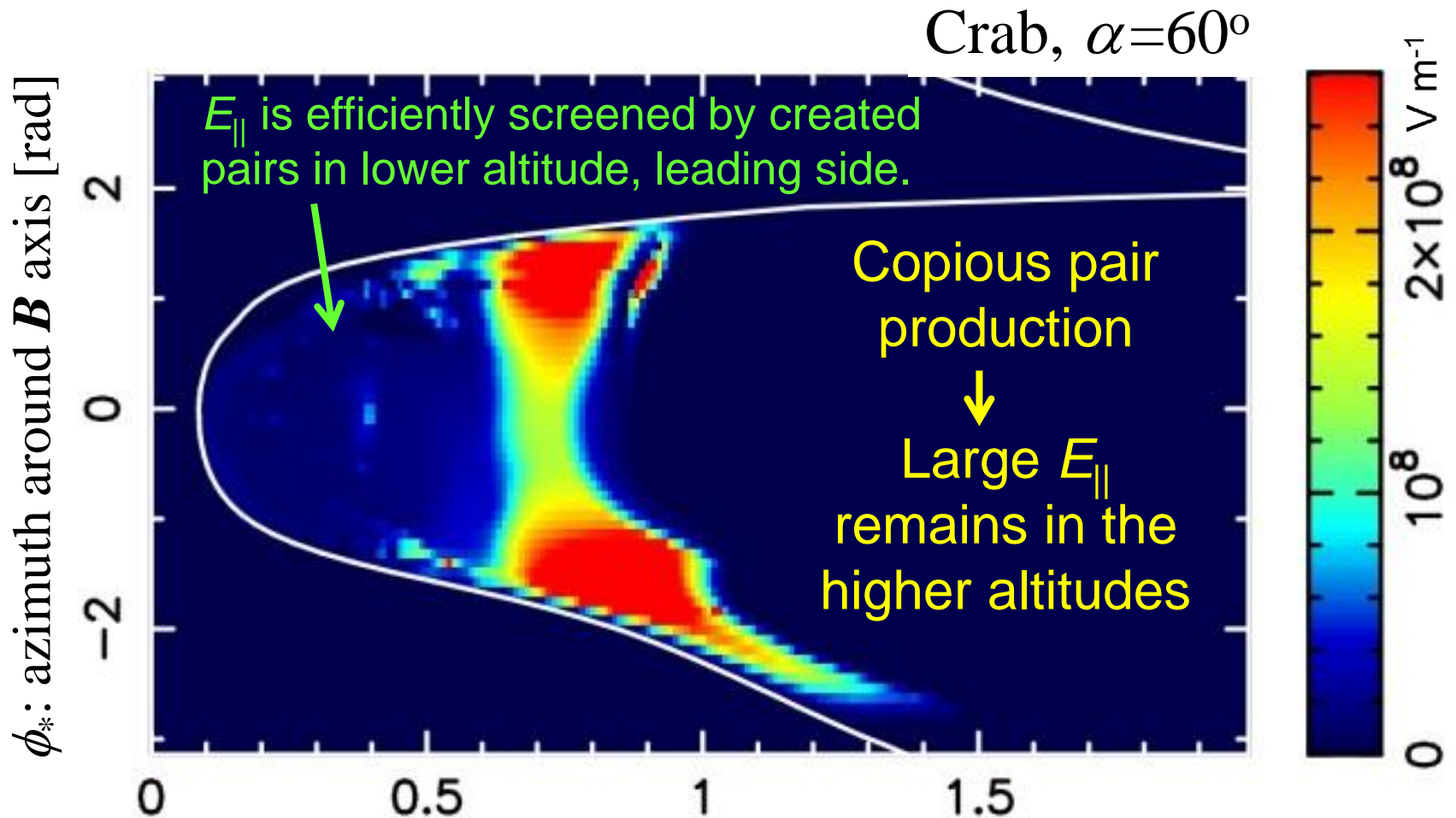
Fractional gap thickness projection on the last-open **B** line surface



§8 HE/VHE Pulsation from the Crab Pulsar

E_{\parallel} is also self-regulated by pair production.

(→ Curvature photon energy changes little for various pulsars.)

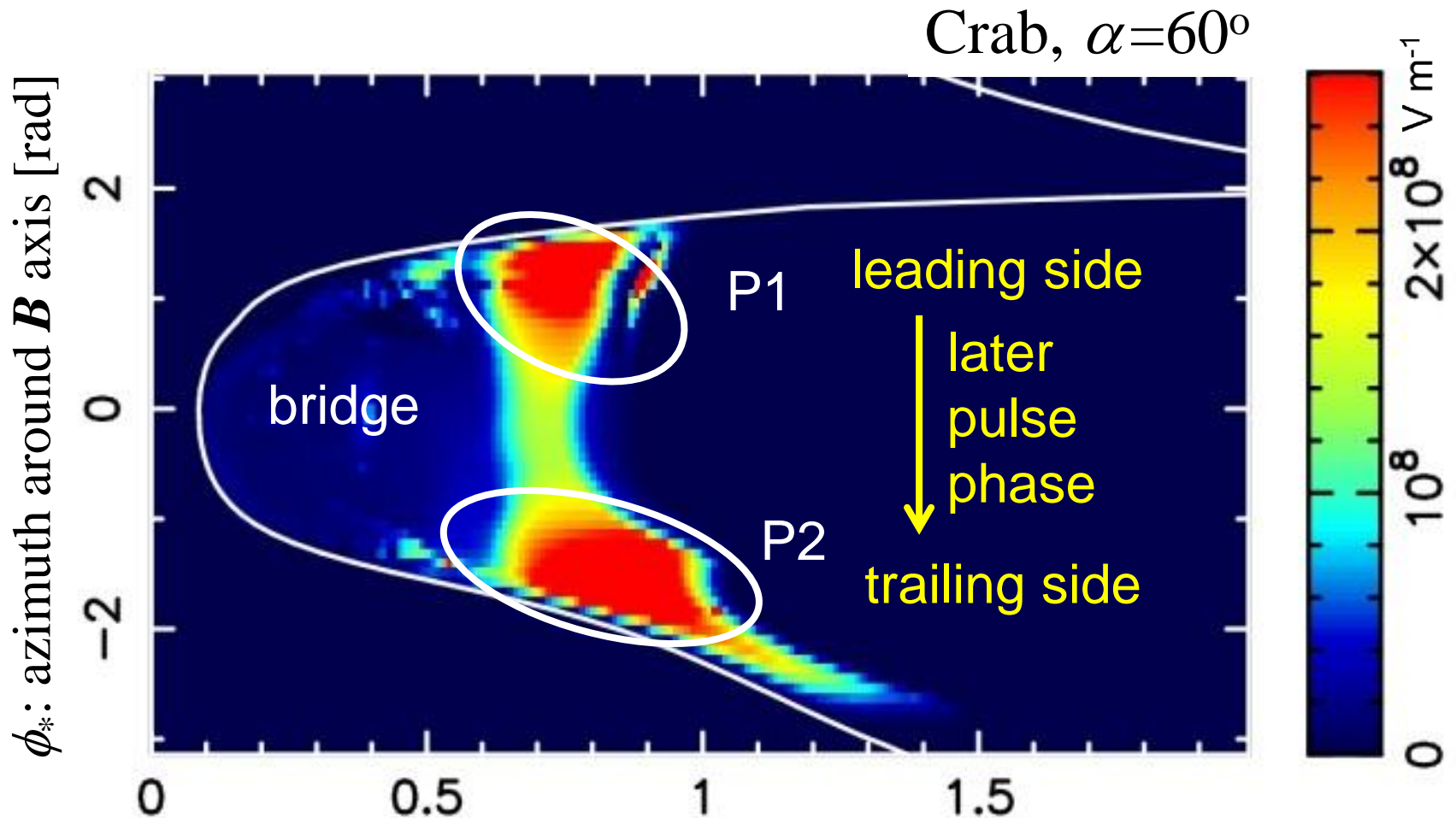


s : distance along B field lines / light-cylinder radius

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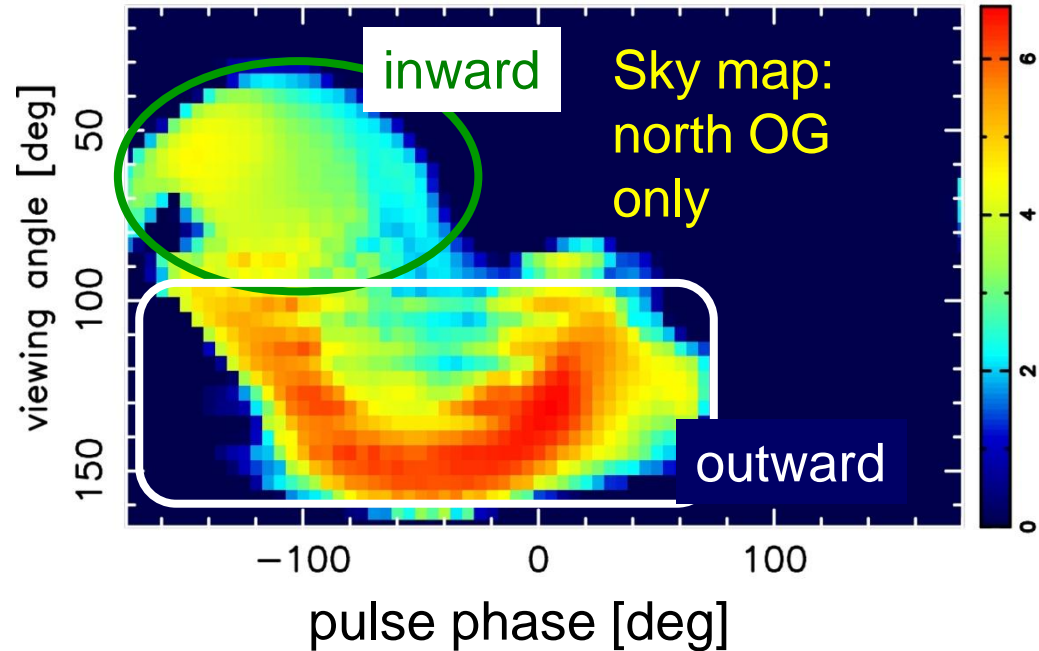
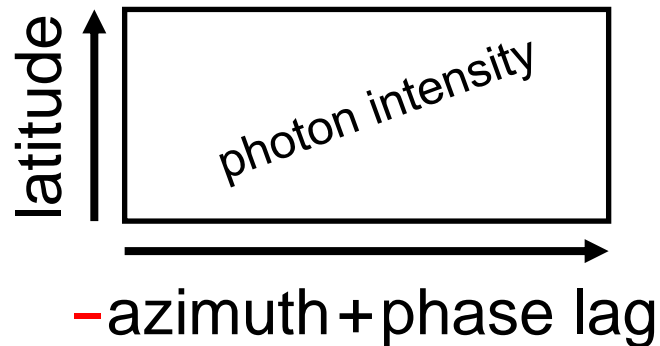
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§8 HE/VHE Pulsation from the Crab Pulsar

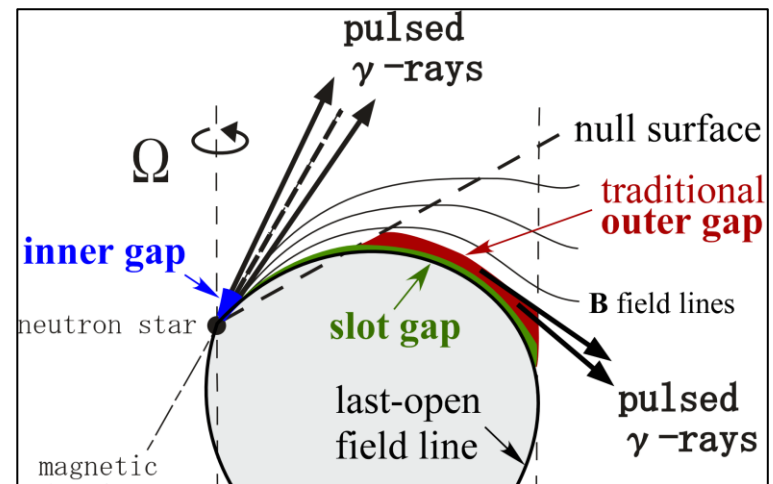
Crab 60°

Using E_{\parallel} , compute emissivity at each position.

Intensity distribution shows **caustic** pattern in the **sky map**.



one NS rotation

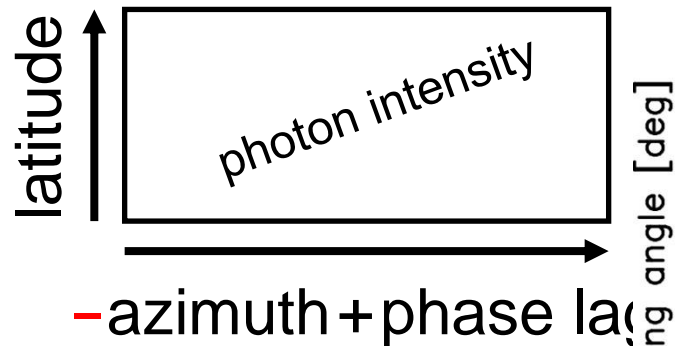


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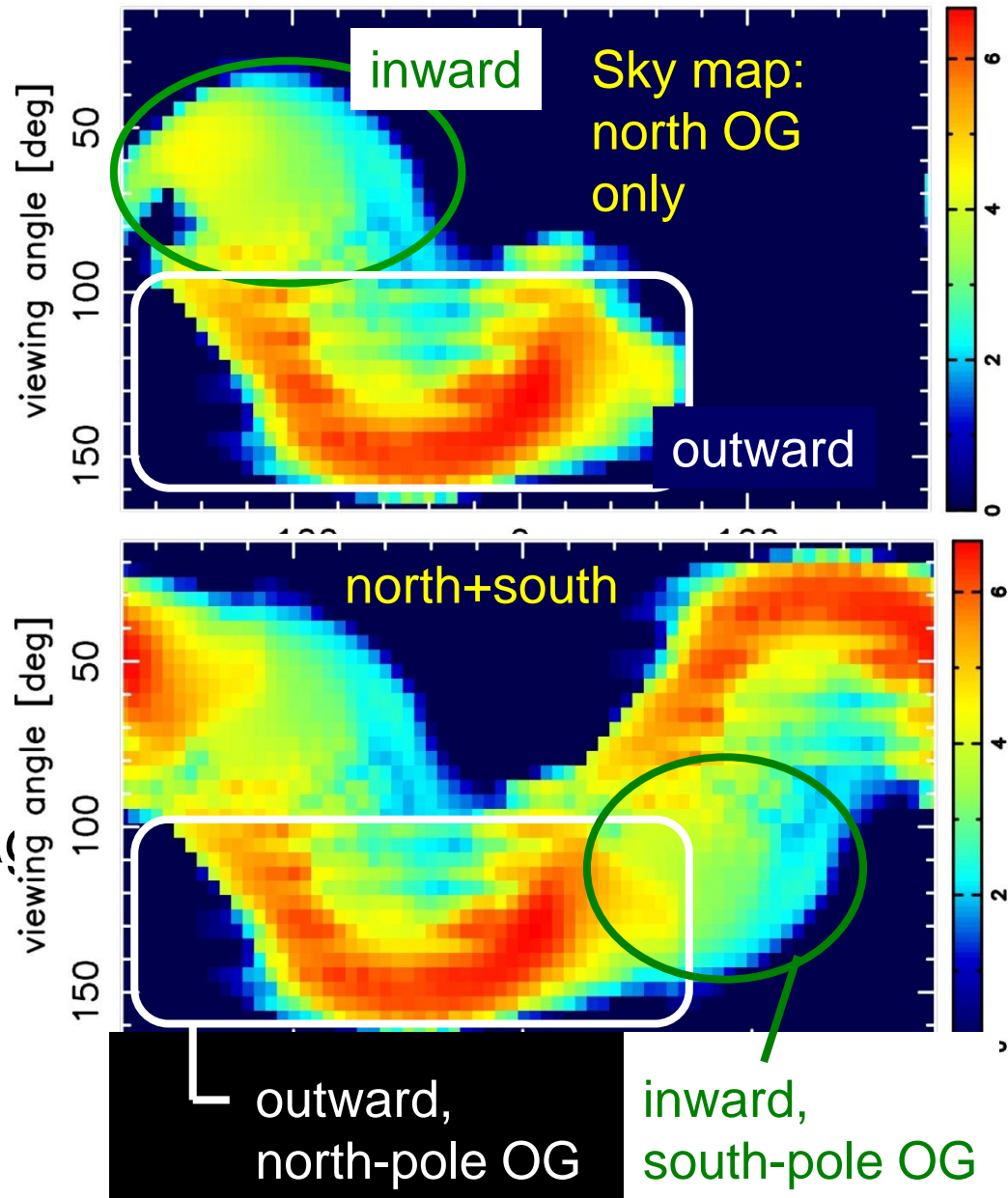
Crab 60°

Using E_{\parallel} , compute emissivity at each position.

Intensity distribution shows **caustic** pattern in the **sky** map.



Consider photons emitted from OGs connected to **both poles**.

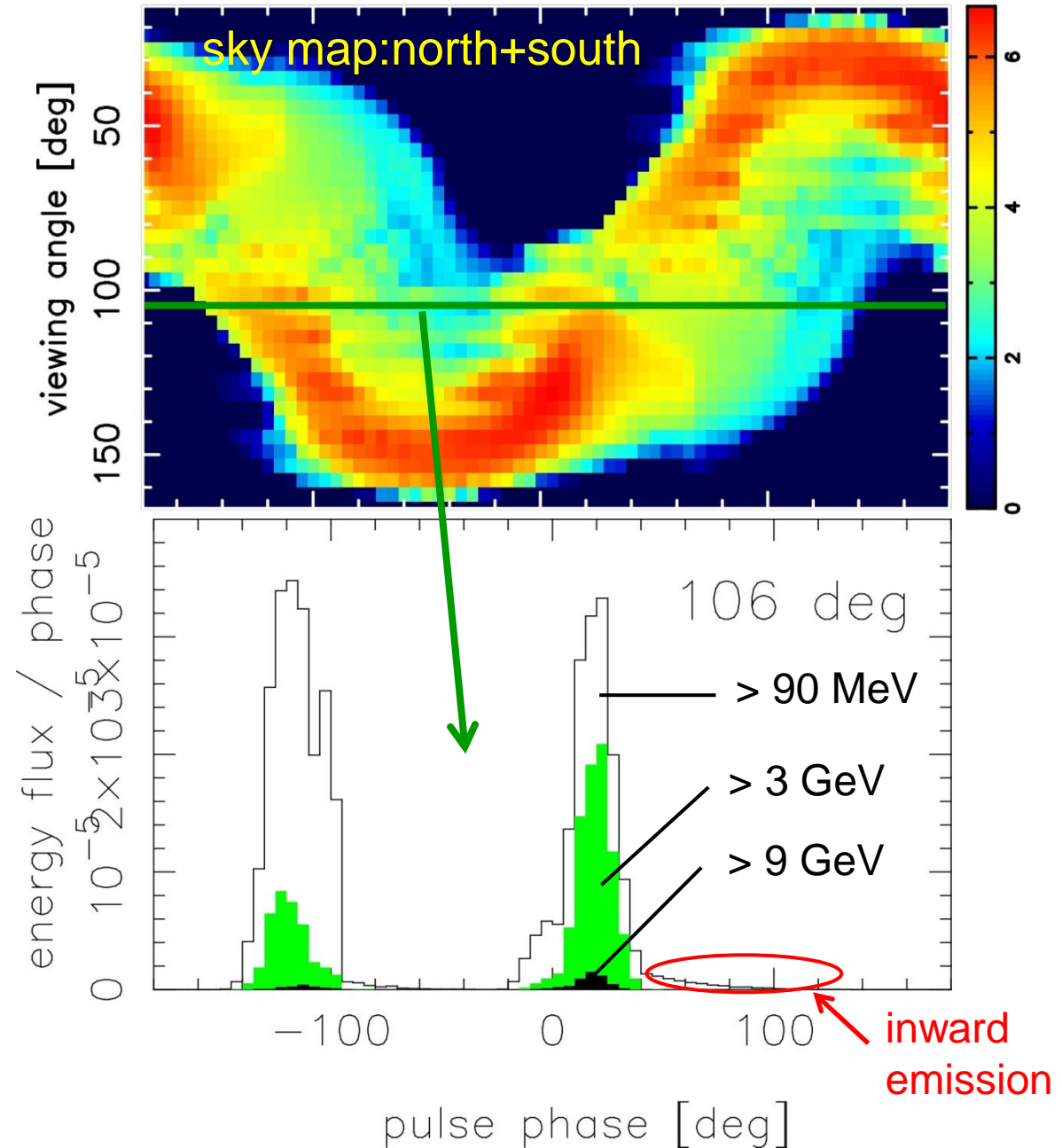


§8 HE/VHE Pulsation from the Crab Pulsar

Crab 60°

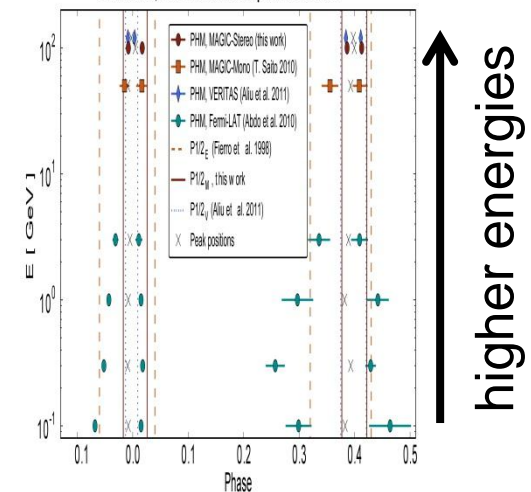
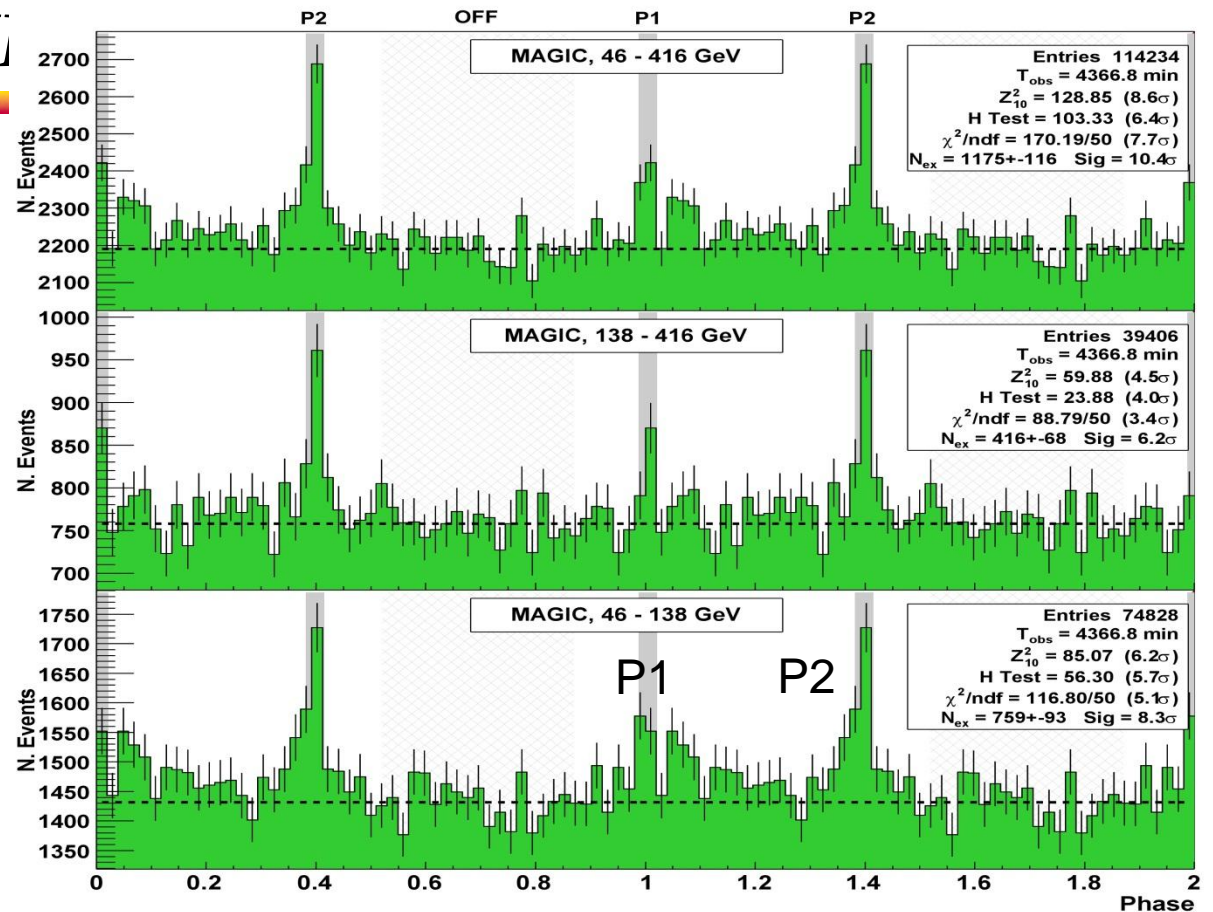
Cut the sky map at a viewing angle ζ to obtain a pulse profile.

With energy-dependent sky map, we obtain pulse profiles at different energies.



§8 HE/VH

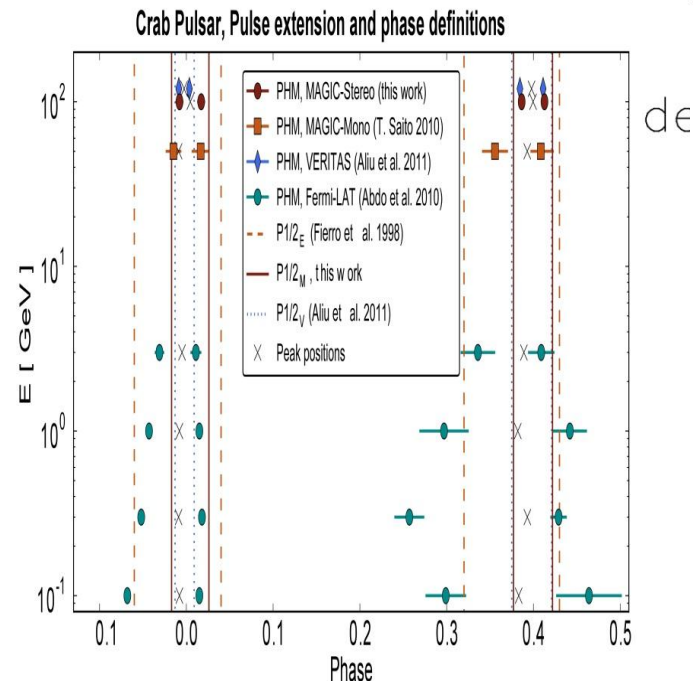
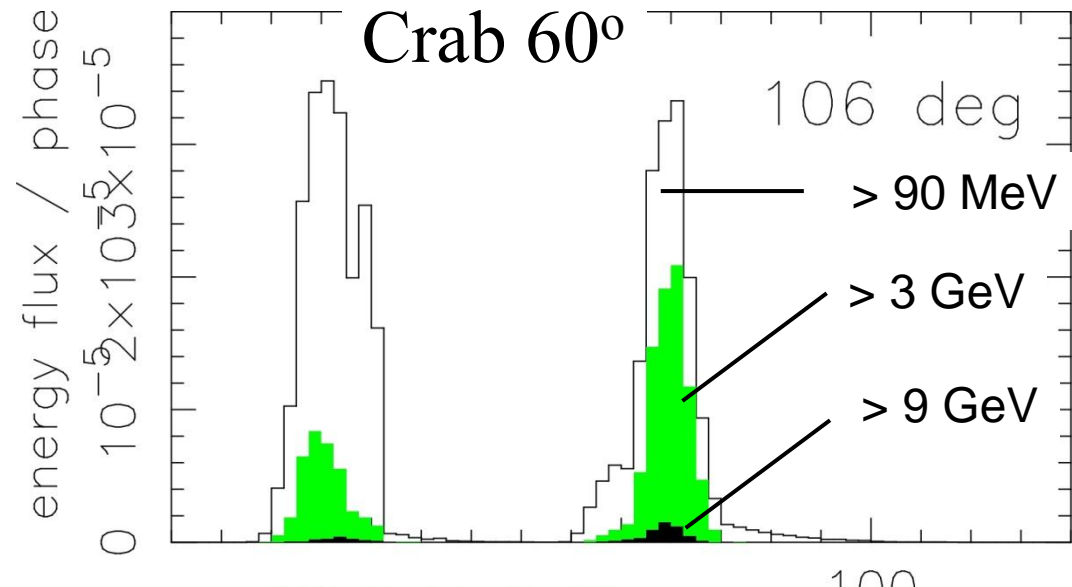
If we look at the details, however, the energy-dependent pulse profile does not reproduce the Fermi and MAGIC observations.



§8 HE/VHE Pulsation from the Crab Pulsar

Some details ...

The **peak width** appears to be roughly **constant** from 0.1 to 10 GeV, while observed peak width sharpens in higher energies.

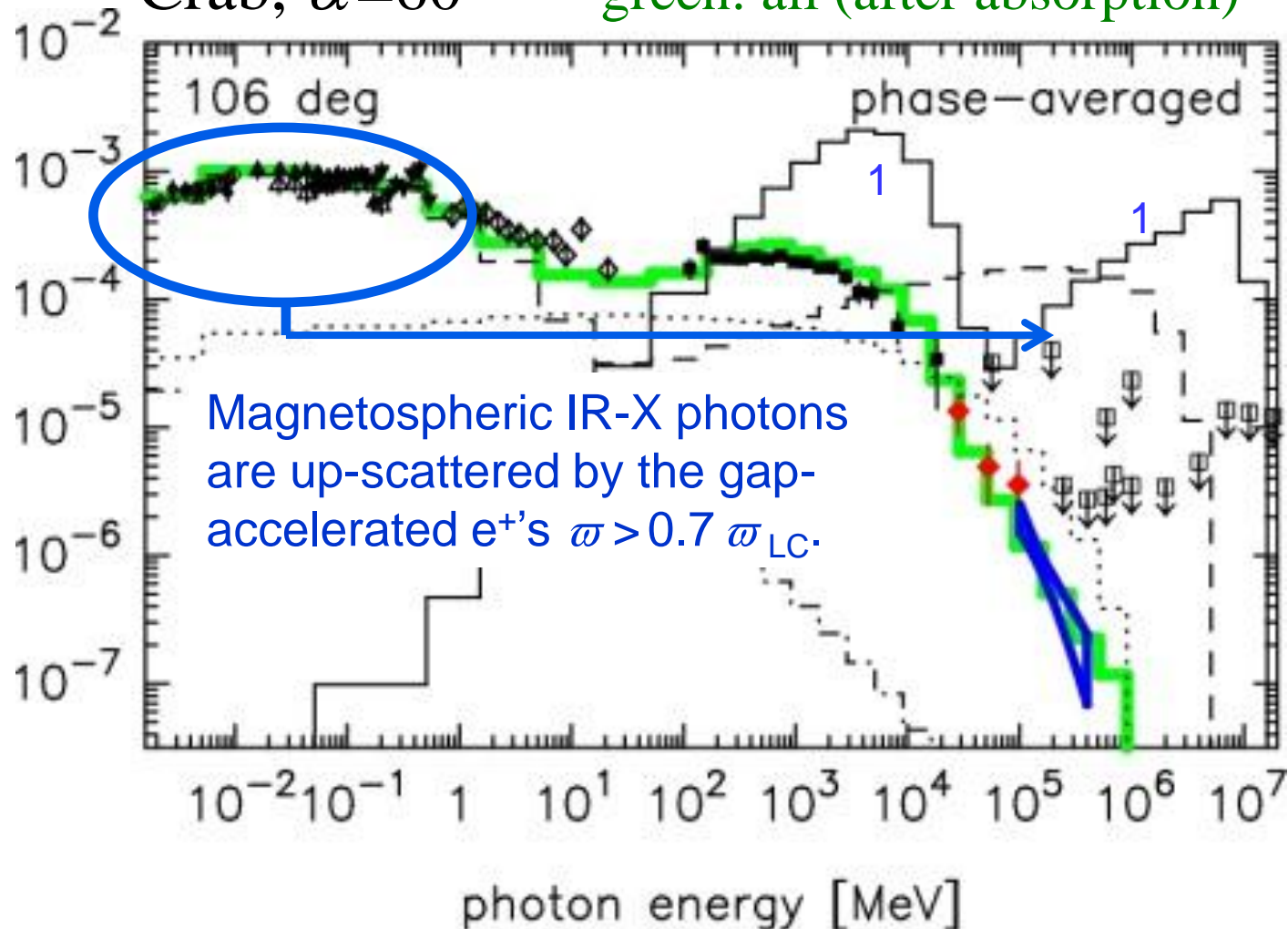


§8 HE/VHE Pulsation from the Crab Pulsar

Phase-averaged spectrum

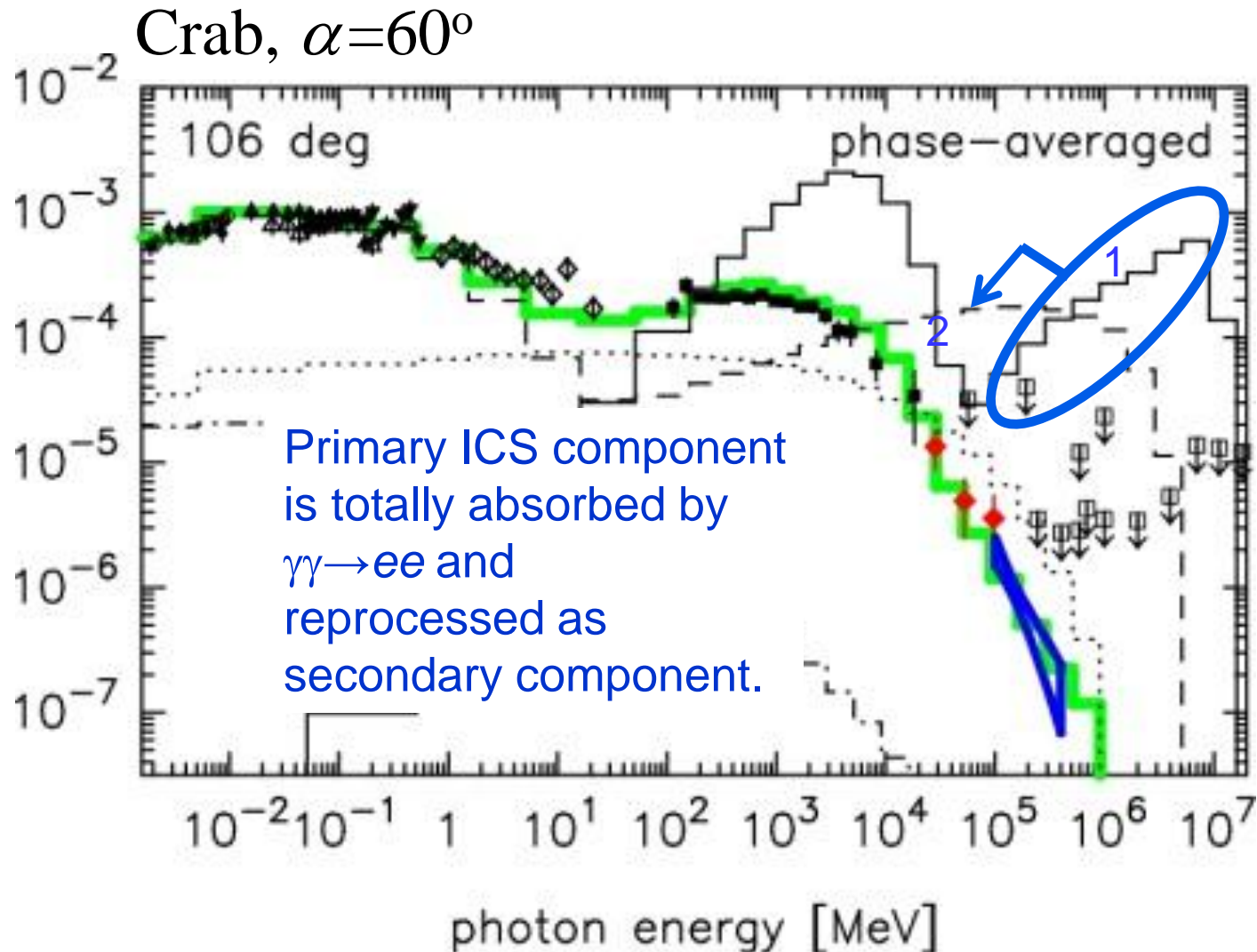
Crab, $\alpha=60^\circ$

solid: primary (bef. absorption)
dashed: secondary (bef. absorption)
dotted: tertiary (bef. absorption)
green: all (after absorption)



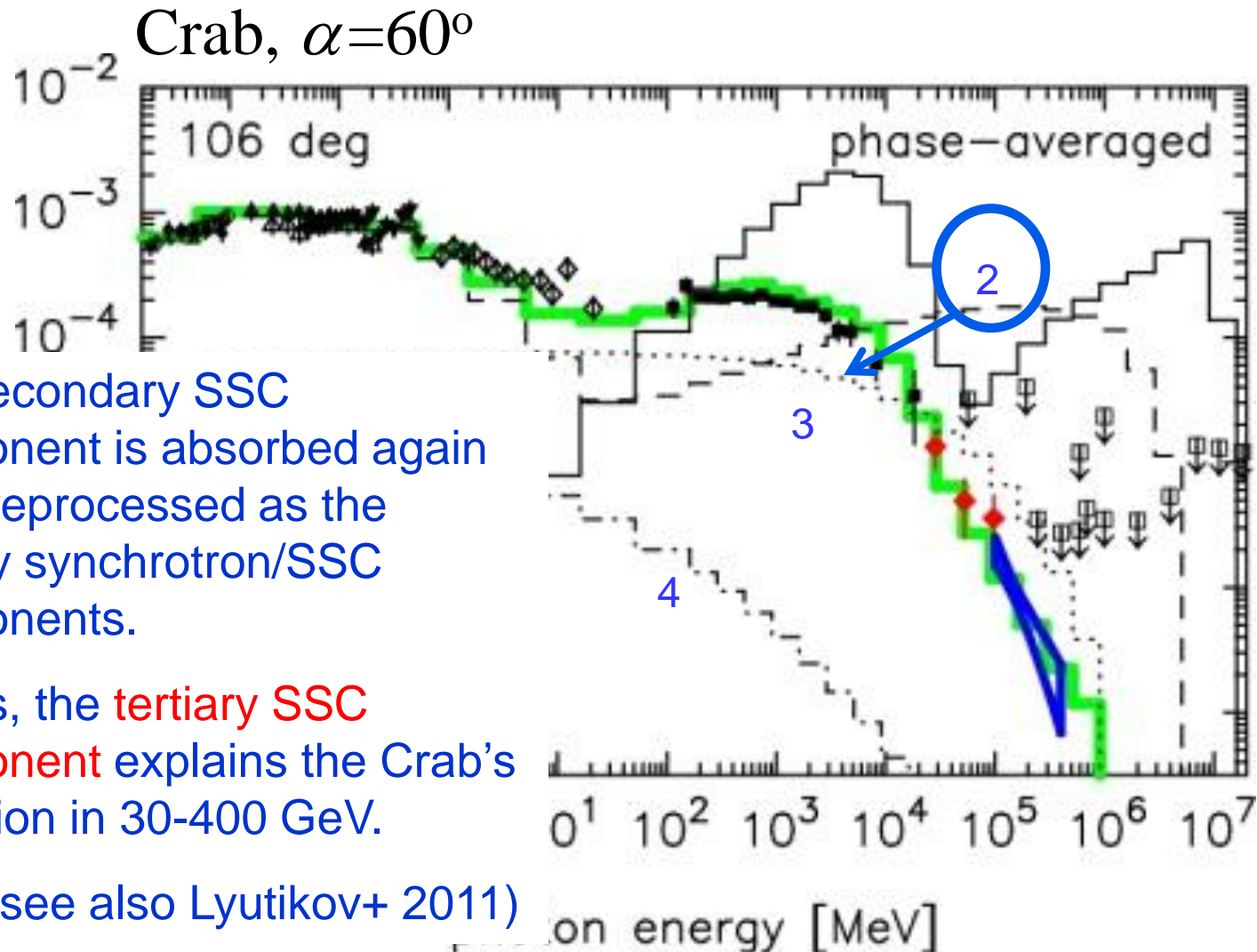
§8 HE/VHE Pulsation from the Crab Pulsar

Phase-averaged spectrum



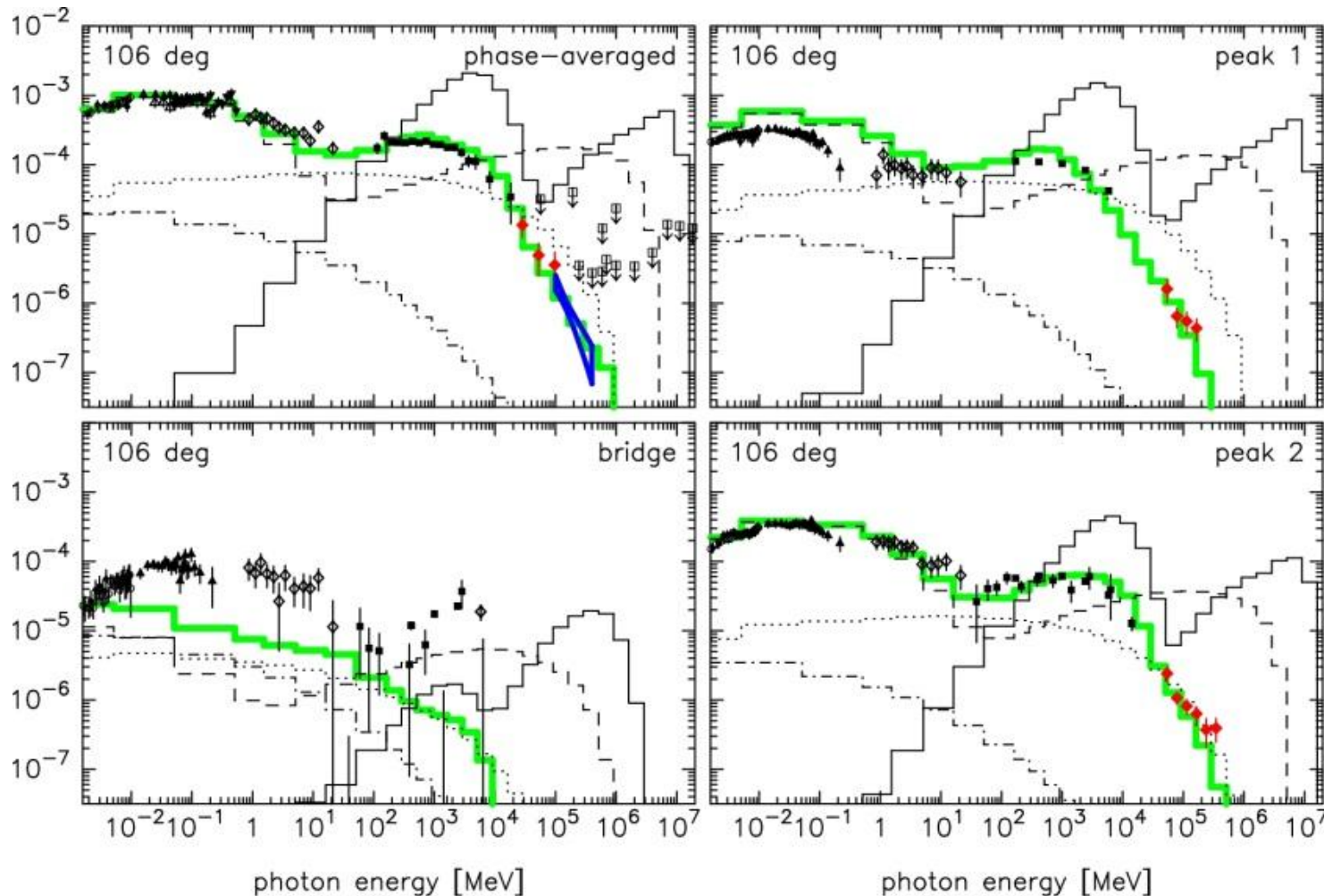
§8 HE/VHE Pulsation from the Crab Pulsar

Phase-averaged spectrum



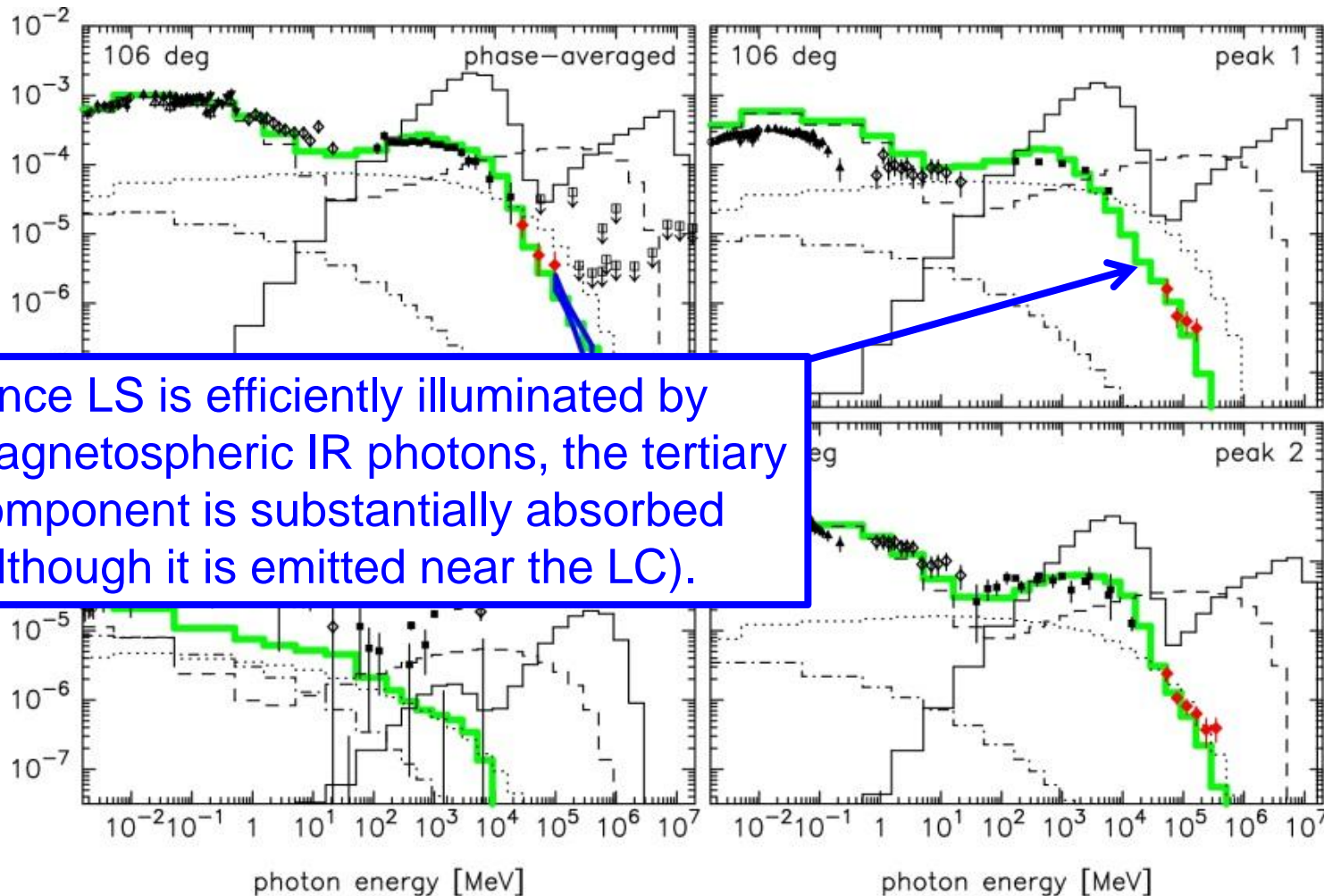
§8 HE/VHE Pulsation from the Crab Pulsar

Phase-resolved spectrum (in LAT-defined phase bins):
Crab, $\alpha=60^\circ$



§8 HE/VHE Pulsation from the Crab Pulsar

Phase-resolved spectrum (in LAT-defined phase bins):
Crab, $\alpha=60^\circ$

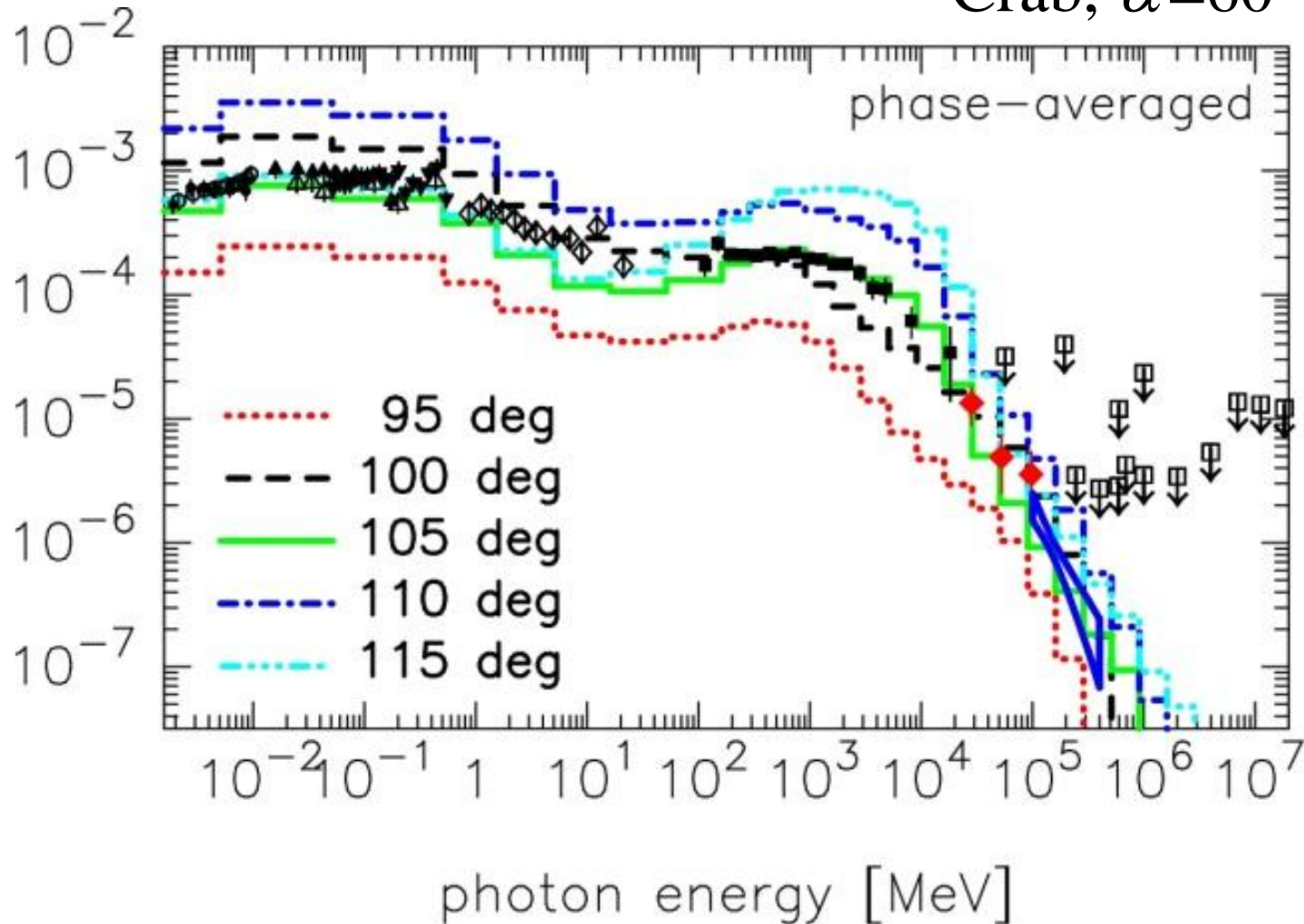


Since LS is efficiently illuminated by magnetospheric IR photons, the tertiary component is substantially absorbed (although it is emitted near the LC).

§8 HE/VHE Pulsation from the Crab Pulsar

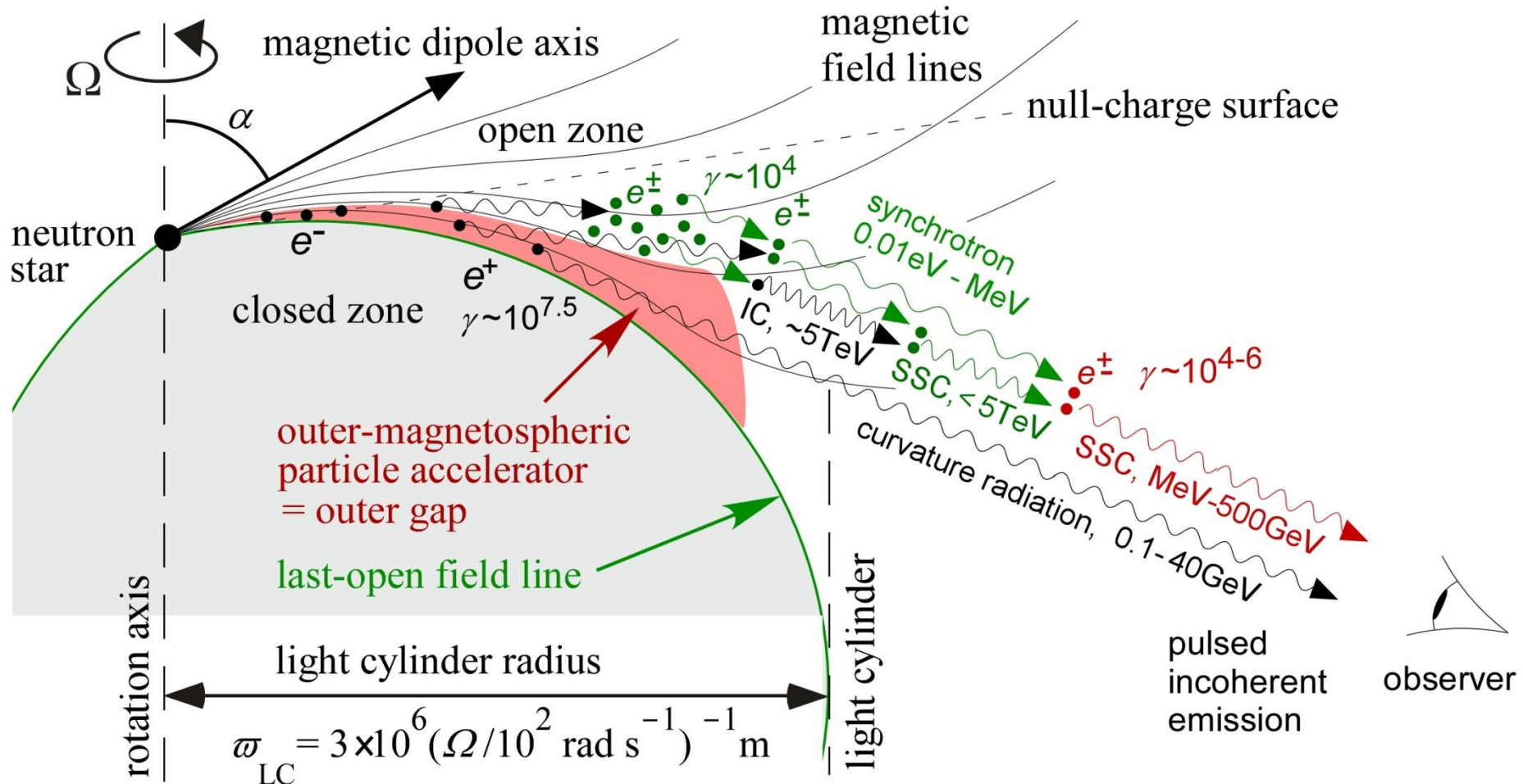
Viewing angle dependence of phase-averaged spectrum:

Crab, $\alpha=60^\circ$



§8 HE/VHE Pulsation from the Crab Pulsar

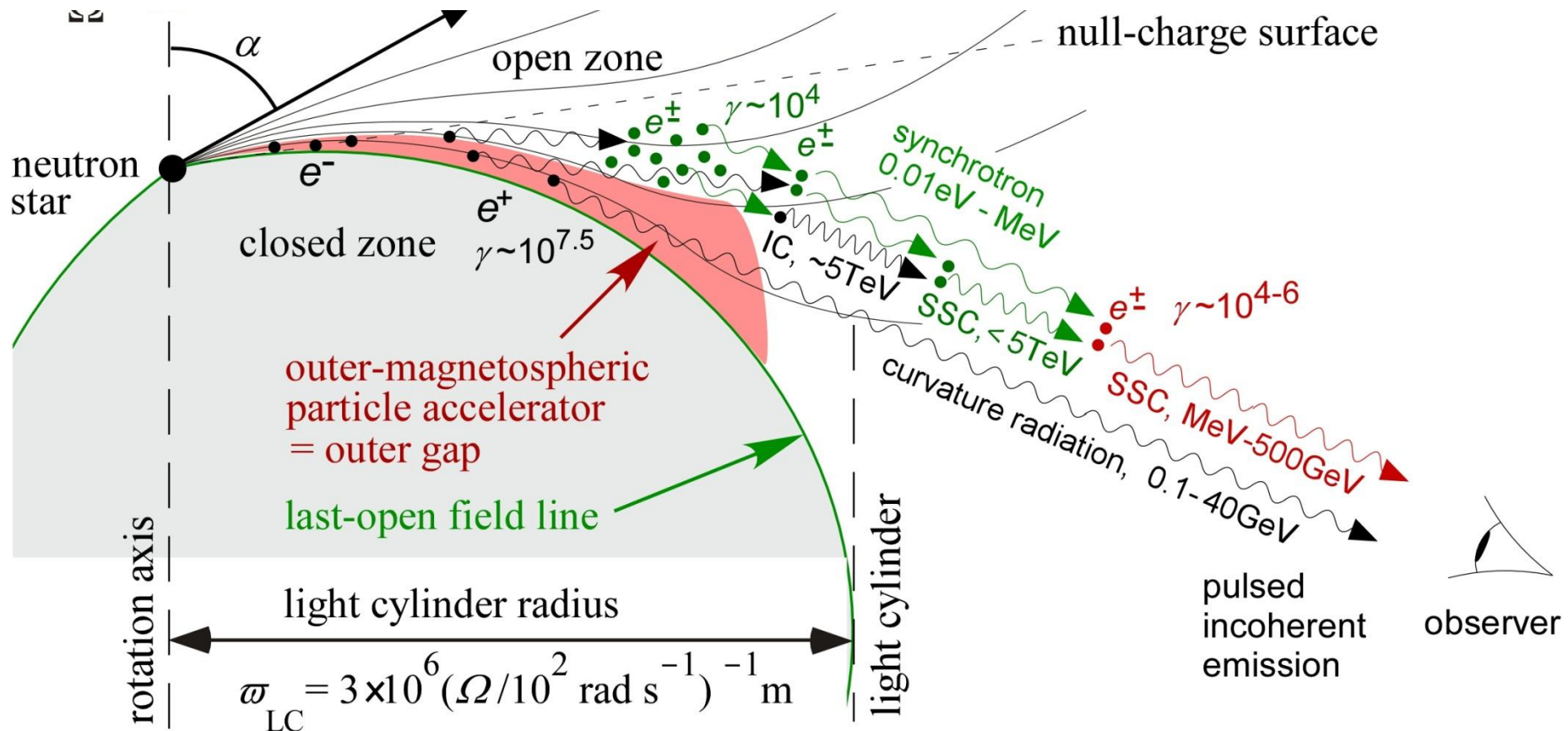
Schematic picture of cascading pairs and their emissions:



§8 HE/VHE Pulsation from the Crab Pulsar

Schematic picture of cascading pairs and their emissions:

Unfortunately, the IC flux is vulnerable to **B** geometry near LC.

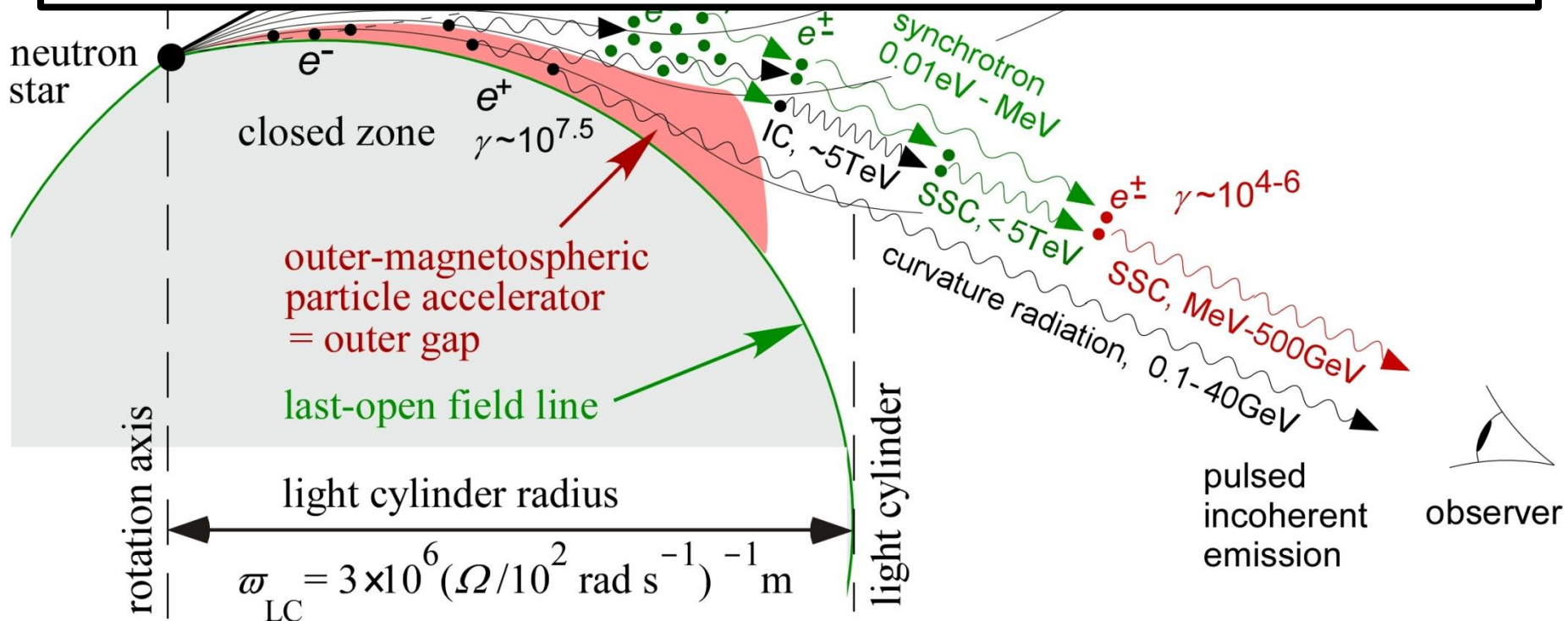


§8 HE/VHE Pulsation from the Crab Pulsar

Schematic picture of cascading pairs and their emissions:

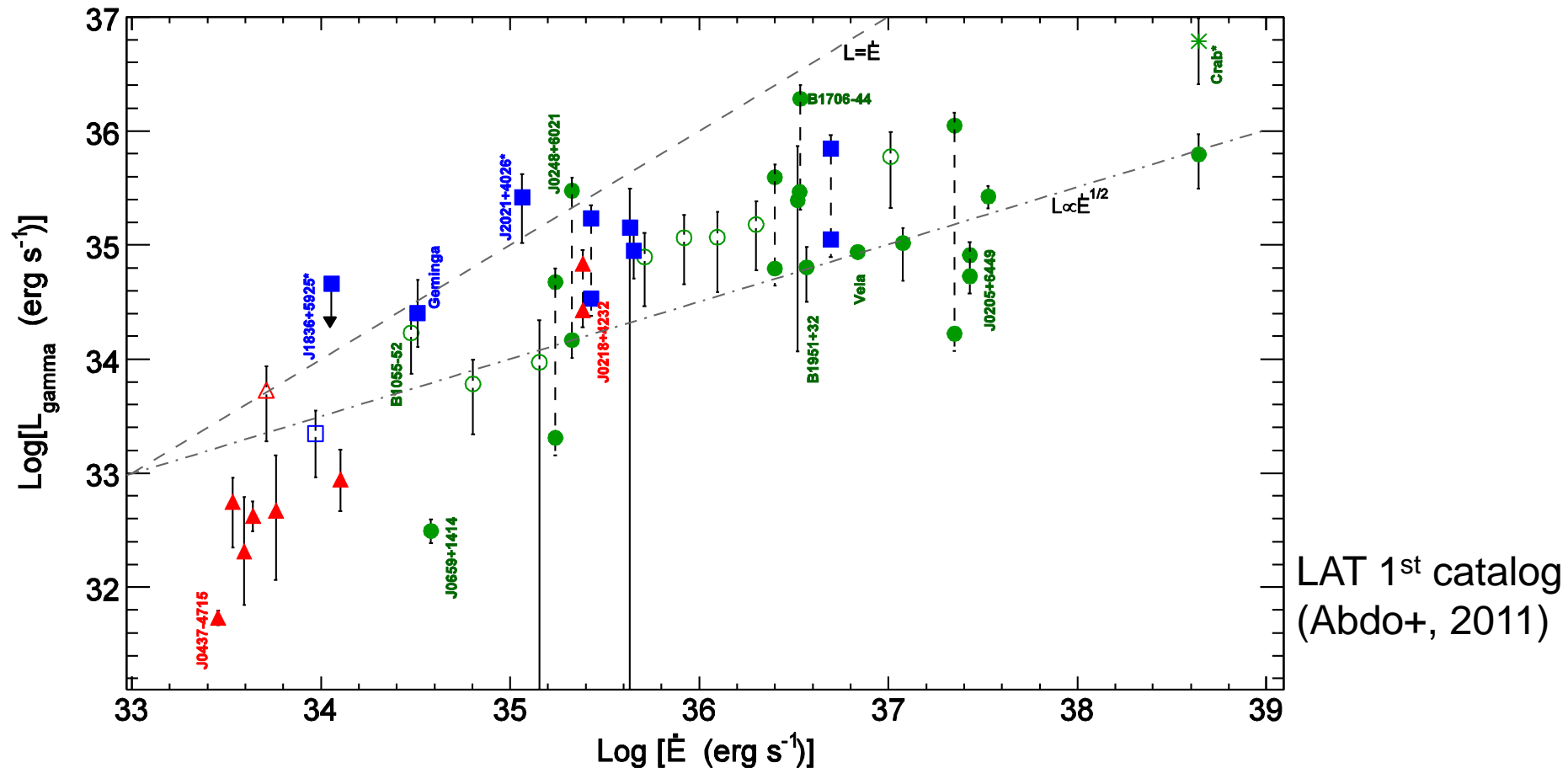
Unfortunately, the IC flux is vulnerable to ***B*** geometry near LC.

Incorporation of **correct *B* geometry near LC** is crucial.



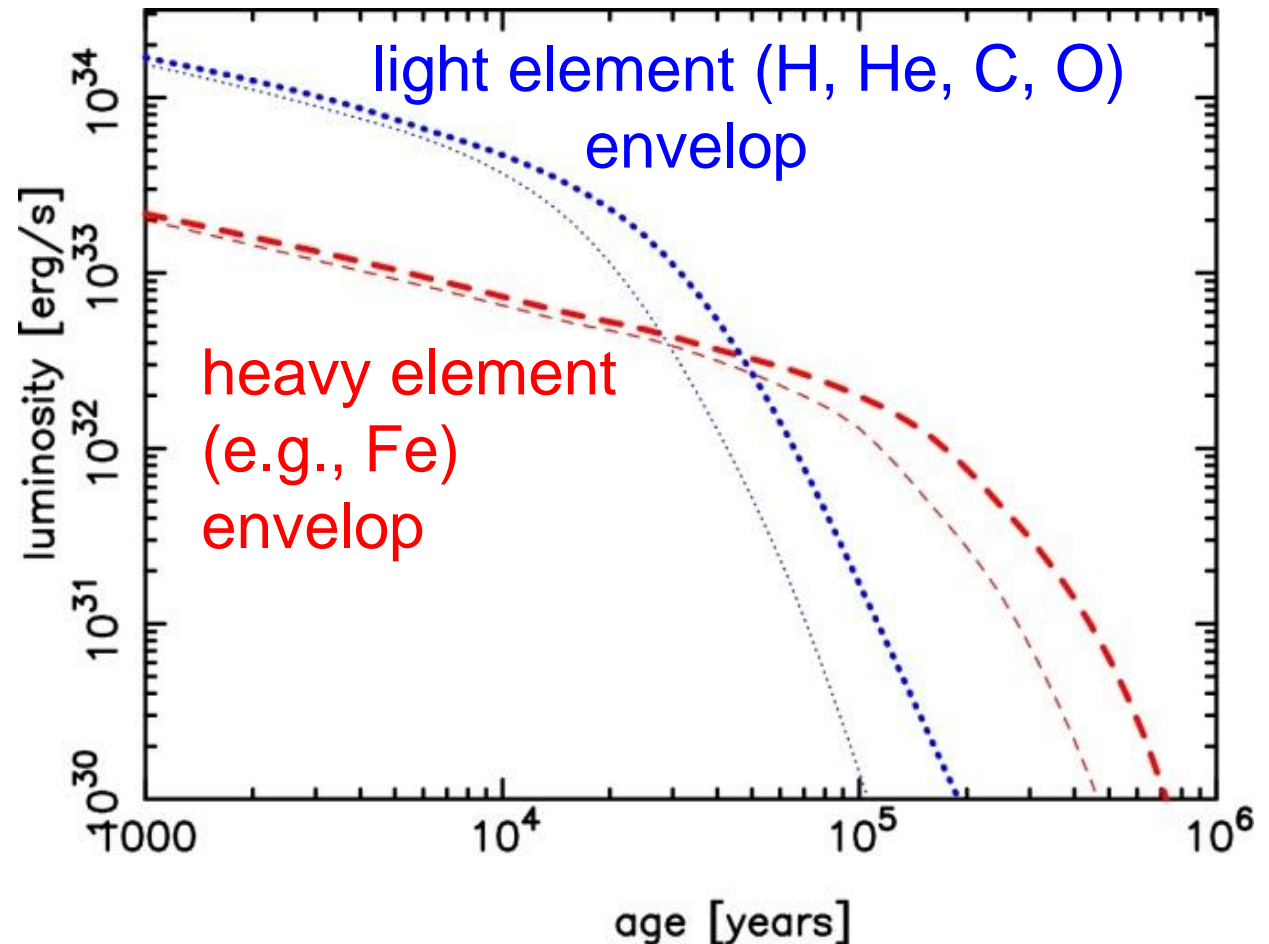
§9 Theoretical derivation of $L_\gamma \sim L_{\text{spin}}^{0.5}$

Let us finally derive the observed relationship,
 $L_\gamma \propto L_{\text{spin}}^{0.5}$, both analytically and numerically.



§9 Theoretical derivation of $L_\gamma \sim L_{\text{spin}}^{0.5}$

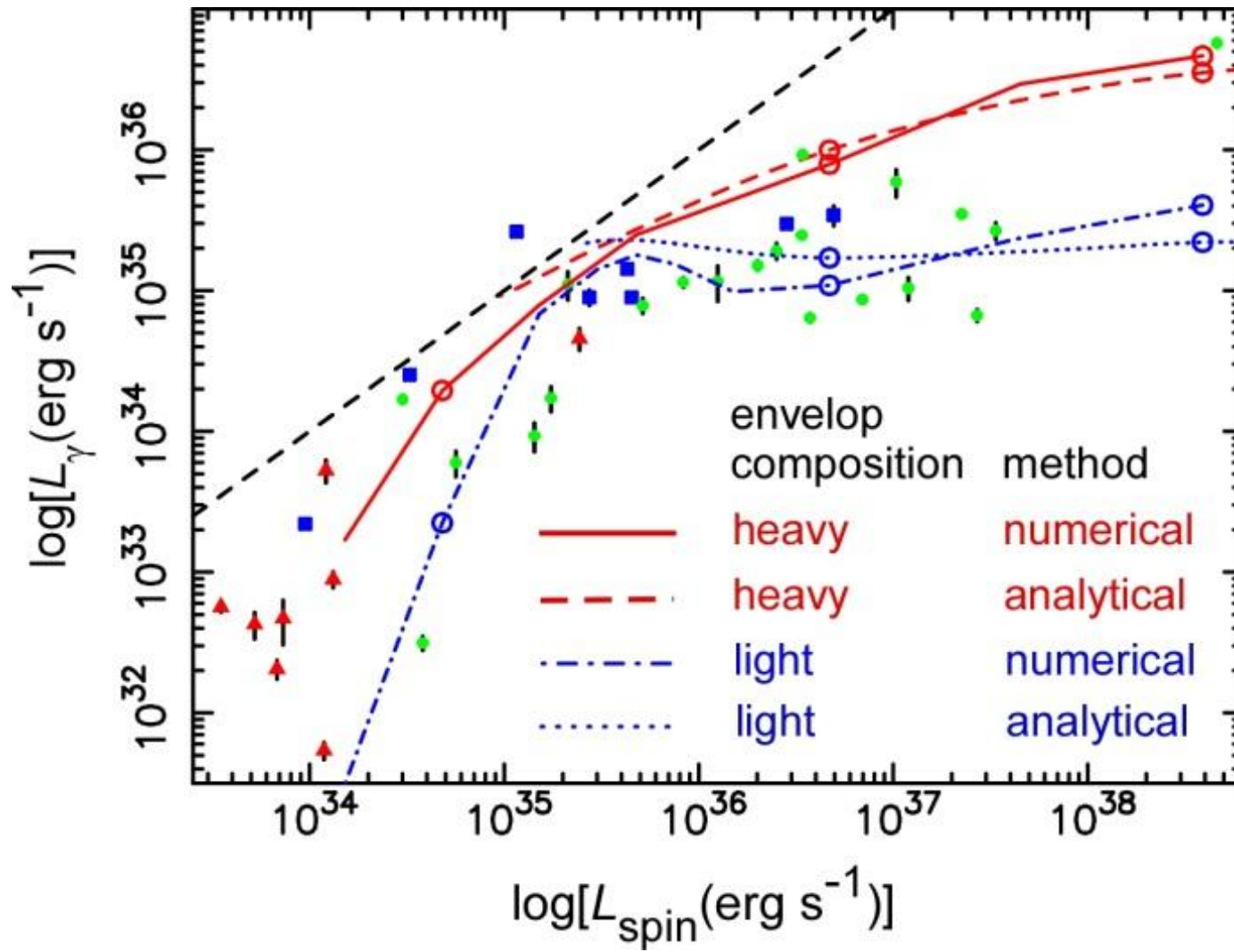
To specify the $\gamma\gamma$ pair production rate, which governs gap evolution, we adopt the minimum cooling scenario.



Minimal cooling
scenario
(Page + 2004)

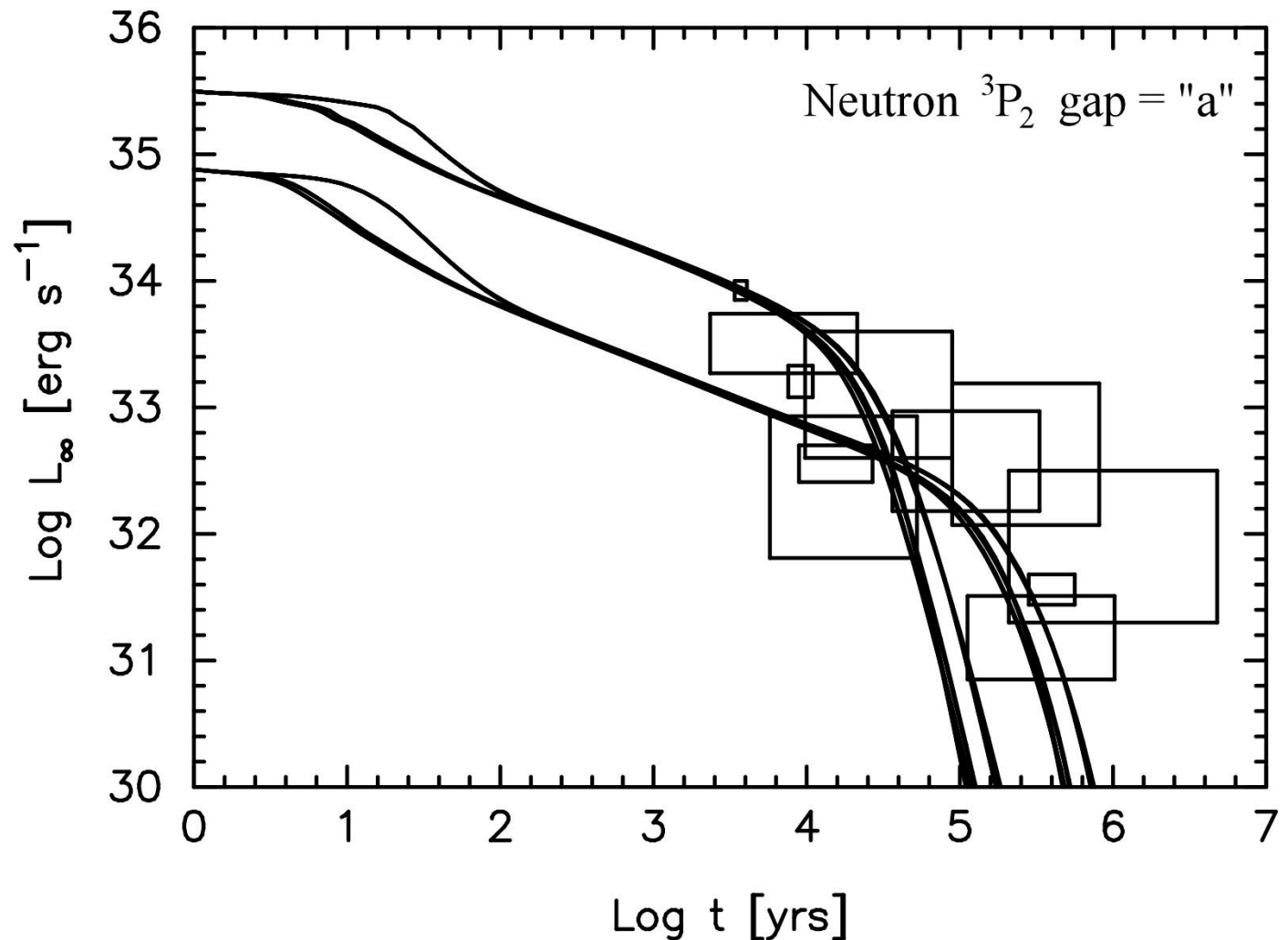
§9 Theoretical derivation of $L_\gamma \sim L_{\text{spin}}^{0.5}$

Gap closure condition, $(N_\gamma \tau)_{\text{in}}(N_\gamma \tau)_{\text{out}} = 1$, gives analytical predictions of L_γ evolution.



§9 Theoretical derivation of $L_\gamma \sim L_{\text{spin}}^{0.5}$

Also derive L_γ evolution **numerically**, considering both **cooling NS emission & heated PC emission**.

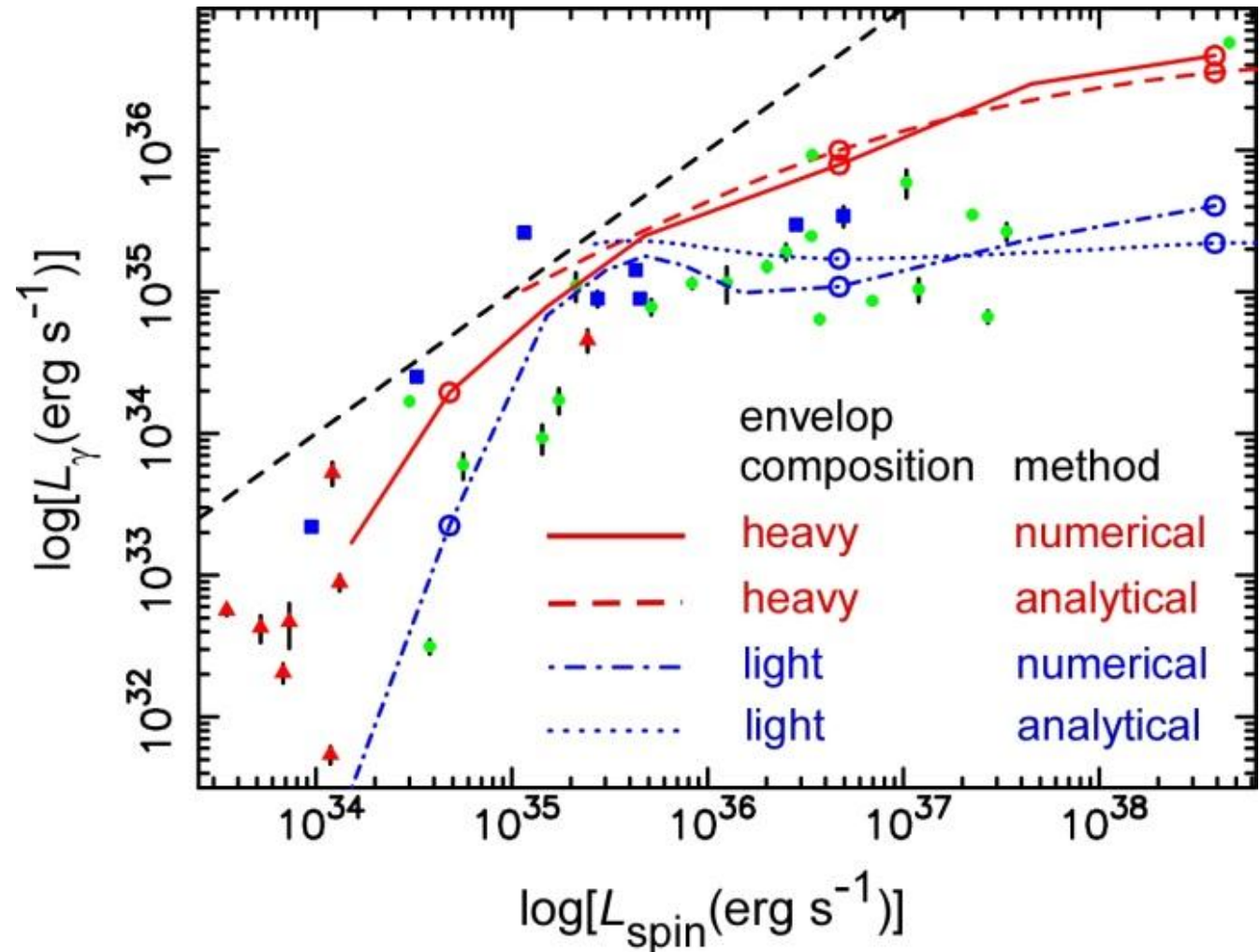


§9 Theoretical derivation of $L_\gamma \sim L_{\text{spin}}^{0.5}$

Numerical solution is consistent with analytical one.

Greater L_γ than analytical ones.

Because emission from higher altitudes (LS:P1 & TS:P2) also contribute, in addition to lower altitudes (bridge).



§9 Theoretical derivation of $L_\gamma \sim L_{\text{spin}}^{0.5}$

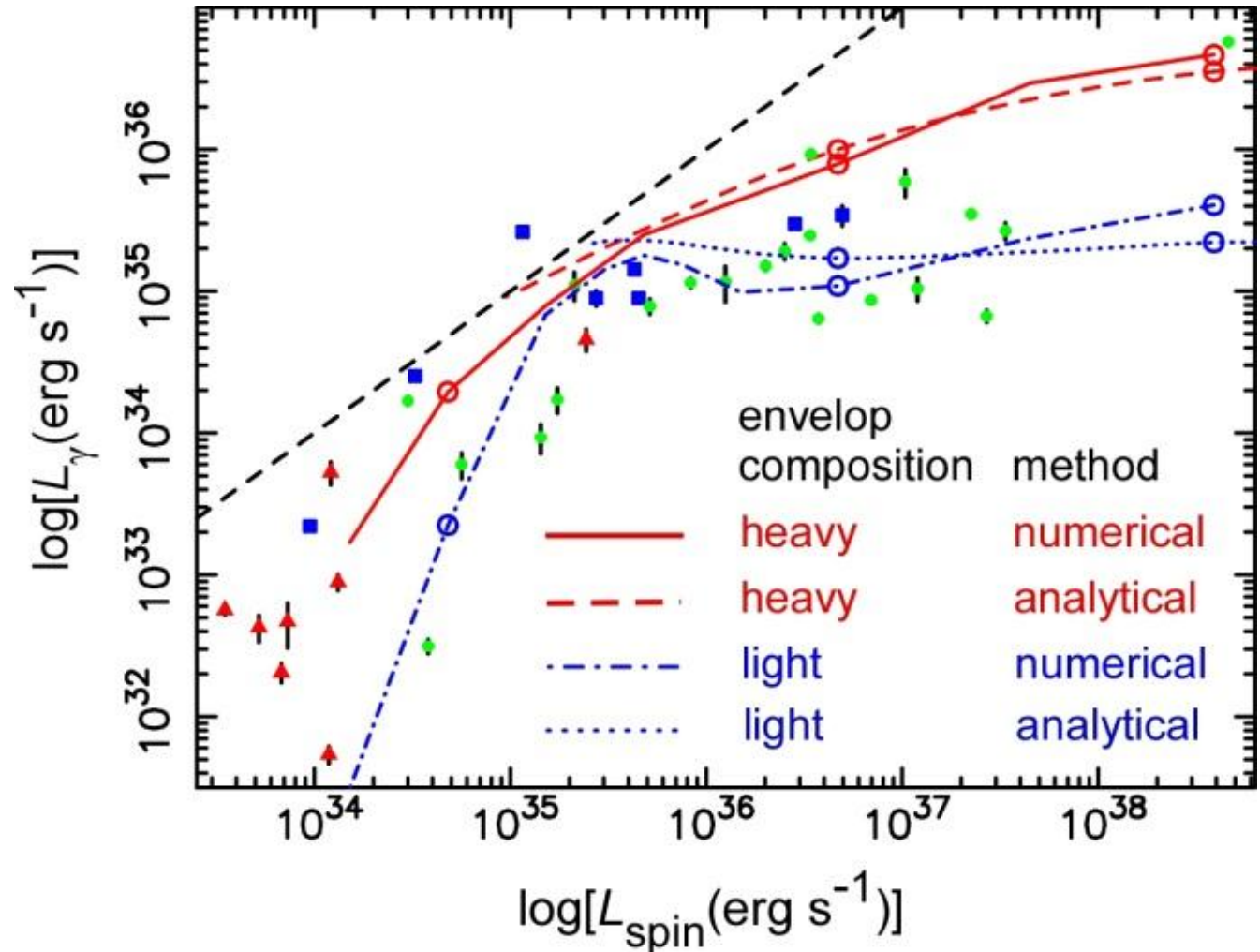
Numerical solution is consistent with analytical one.

ρ declines at
 $L_{\text{spin}} < 10^{35} \text{ erg/s}$



L_γ declines
sharply.

For **light element**
envelop, L_γ
peaks around
30000 yrs, as
predicted by
analytical
calculation.

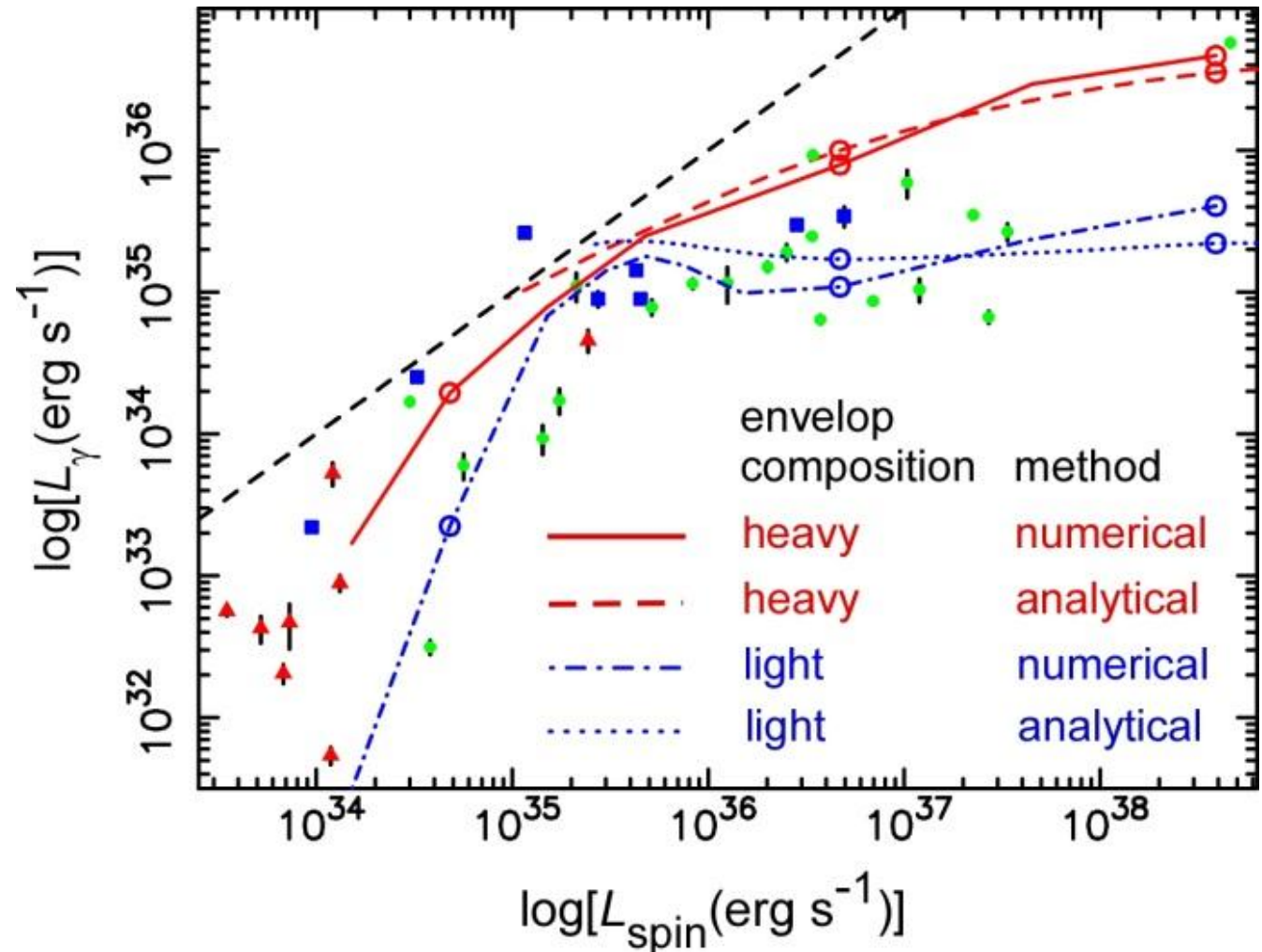


§9 Theoretical derivation of $L_\gamma \sim L_{\text{spin}}^{0.5}$

Numerical solution is consistent with analytical one.

A realistic NS will have envelop composition between the two extreme cases, light and heavy element envelops.

Thus, the actual L_γ will distribute between the red and blue curves.



Summary

- We can now solve pulsed high-energy emissions from the set of Maxwell ($\text{div}\mathbf{E}=4\pi\rho$) and Boltzmann eqs., if we specify P , dP/dt , α_{incl} , kT_{NS} . We no longer have to assume the gap geometry, E_{\parallel} , e^{\pm} distribution functions.
- By SSC of secondary/tertiary pairs, Crab pulsar's total and phase-resolved spectrum shows a power-law-like shape.
- Observed relationship, $L_{\gamma} \propto L_{\text{spin}}^{0.5}$, can now be derived both analytically and numerically under the OG model.